CARNEGIE MATH PATHWAYS

Contextualized Lesson Design Principles: A Look at the Research

Session 4

C2L PD Workshop

Madison Area Technical College

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Carnegie Foundation for the Advancement of Teaching

What Do We Want Our Students to Learn?

Instructional Goal

To optimize students engagement in and learning of mathematics in a disciplinary context as well as relevant aspects of the discipline itself

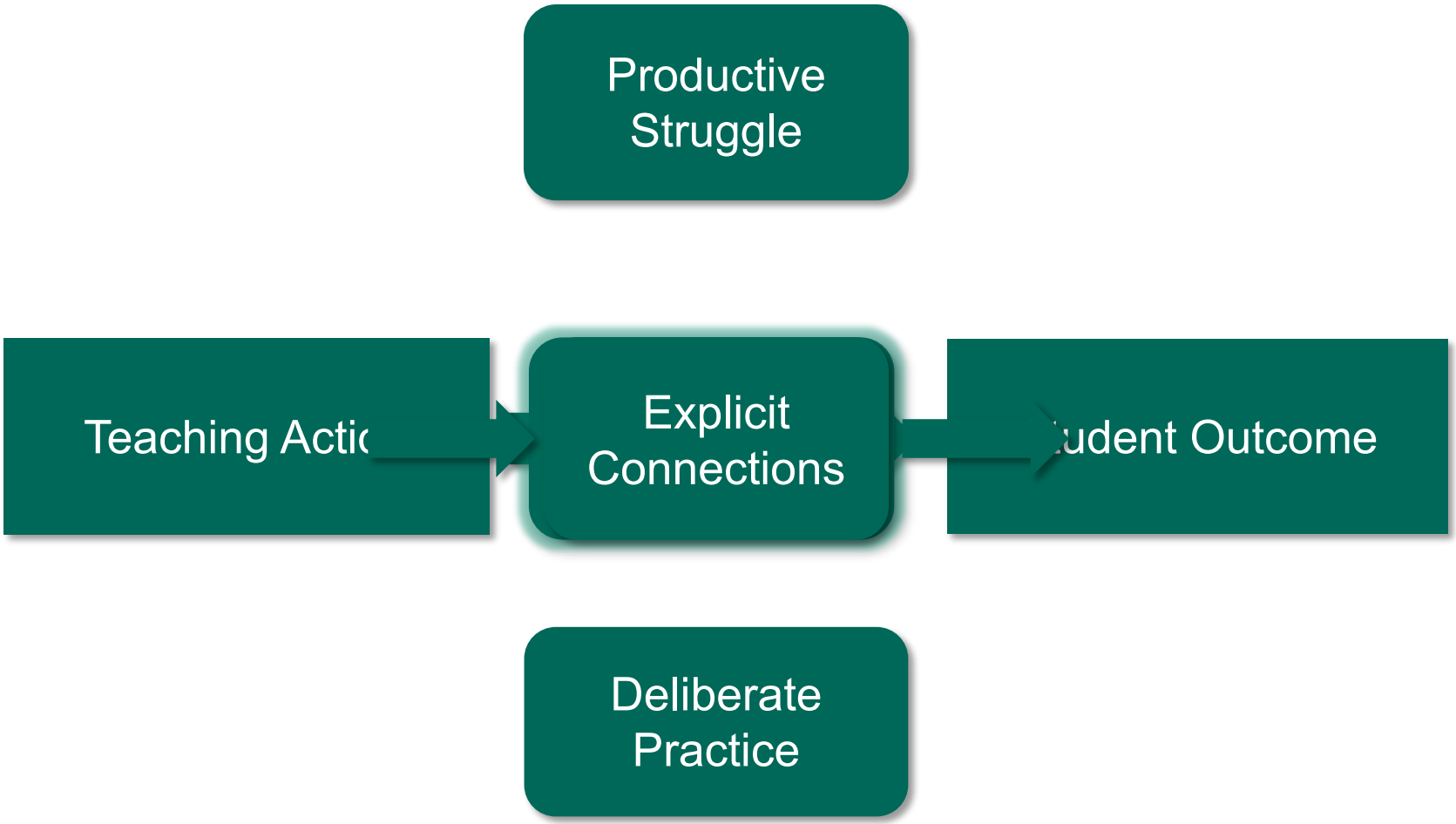
Requires alignment of curricular design and instructional practice

What Do We Want Our Students to Learn?

- *Flexible* vs. *routine* expertise

  (Hatano & Inagaki, 1986)
- What is flexible expertise?
  - Procedural fluency
  - Conceptual understanding
  - Disposition to think/make sense of mathematics
  - Ability to nimbly bring knowledge to bear across a wide array of new situations
  - *Understand how mathematics and the disciplinary context are connected and how to use mathematics in those contexts*

How Do Students Develop Flexible Expertise?



Learning Opportunities

To achieve flexible expertise, students need recurring and sustained opportunities for:

- **Productive struggle** – with important mathematics
- **Explicit connections** – between concepts, procedures, problems, situations
- **Deliberate practice** – increasing variation and complexity over time

Productive Struggle (Hiebert & Grouws, 2007)

We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent.

We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems.

We do not mean the feelings of despair that some students can experience when little of the material makes sense.

The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed.

Desirable Difficulties *(related to struggle)*

- Conditions of instruction that make performance improve rapidly often fail to support long-term retention and transfer
- Conditions of instruction that appear to create difficulties for the learner, slowing the rate of *apparent* learning, often optimize retention and transfer
  - Blocked vs. random practice
  - Studying using multiple-choice vs. short-answer prompts
  - Beanbag study (practice at tested distance vs. tested distance ±1 ft.)

What does productive struggle look like?

- Classroom might be quiet at first, but then it may explode into lively discussion within several small groups.
- During the small group discussions, students might try out ideas with each other, critique those ideas, and together build/create different ideas and understandings that help move the understanding of the problem forward.
- The nature of the discussion about the problem would likely be more about approaches and concepts than on procedures.
- The instructor circulates around the room listening to students and … asking questions and probing for clarification of the students' thinking.
- The instructor also keeps the students from heading too far down an unproductive path by carefully asking questions that might cause the students to reconsider their approach or by challenging them with discrepant evidence that prompts such a reconsideration.
- The instructor usually doesn't explicitly offer answers or directions, but rather offers comments designed to keep student thinking proceeding productively.

How do we promote productive struggle?

- Task is genuinely motivating to solve.
- Task is often complex (though not necessarily complicated).
- Task is likely to produce multiple student approaches/responses.
- Task is slightly beyond immediate comprehension for your students.
- Instructor should be well aware of the mathematical (and disciplinary) learning goals of the task/lesson and their connections.
- Instructor should prepare by anticipating students' responses and approaches to the task.

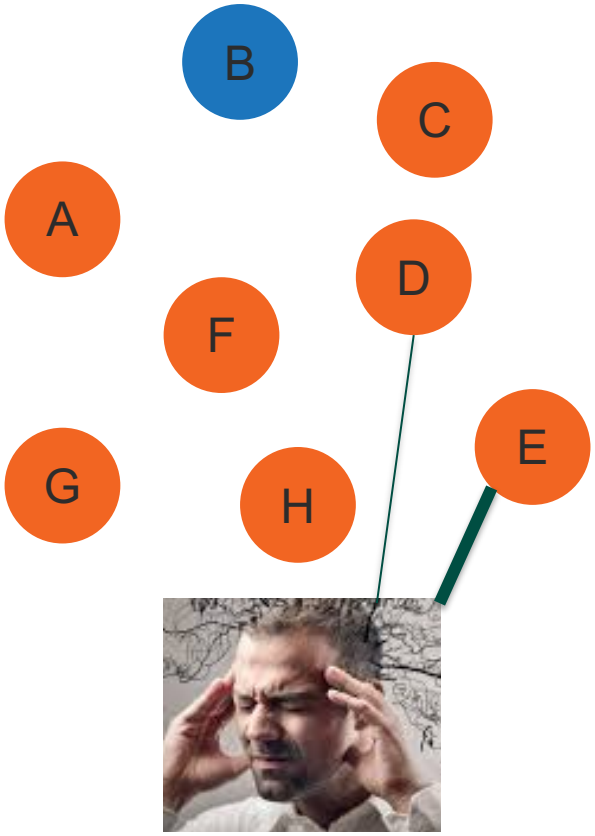
Explicit Connections

- Connections among mathematical/statistical facts, procedures, and ideas, addressed in an explicit and public way.
- The focus should be on making sense of the concepts at hand so that the correctness of a solution is not based on the answer key but on whether the solution makes sense.

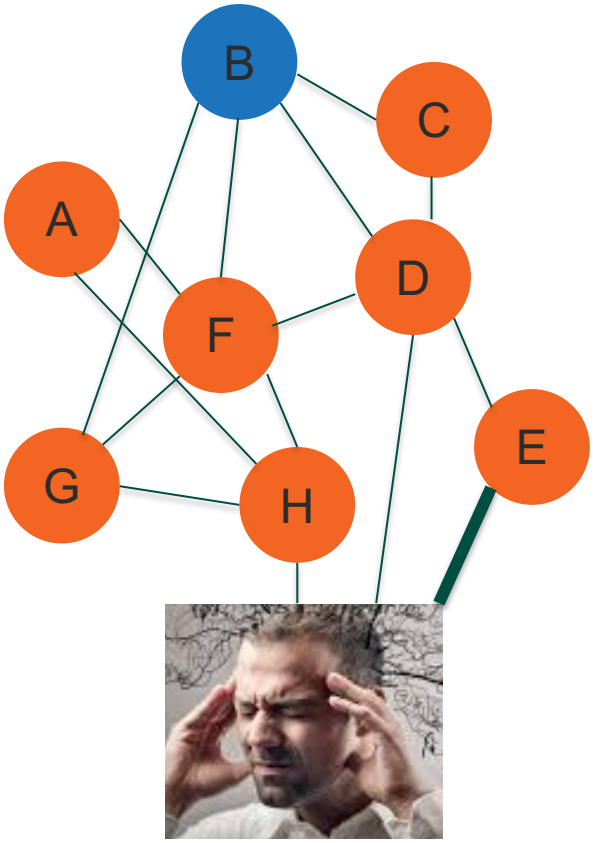
Thinking Hard

The Power of Connections

Few Connections



Many Connections



Explicit Connections

Goals

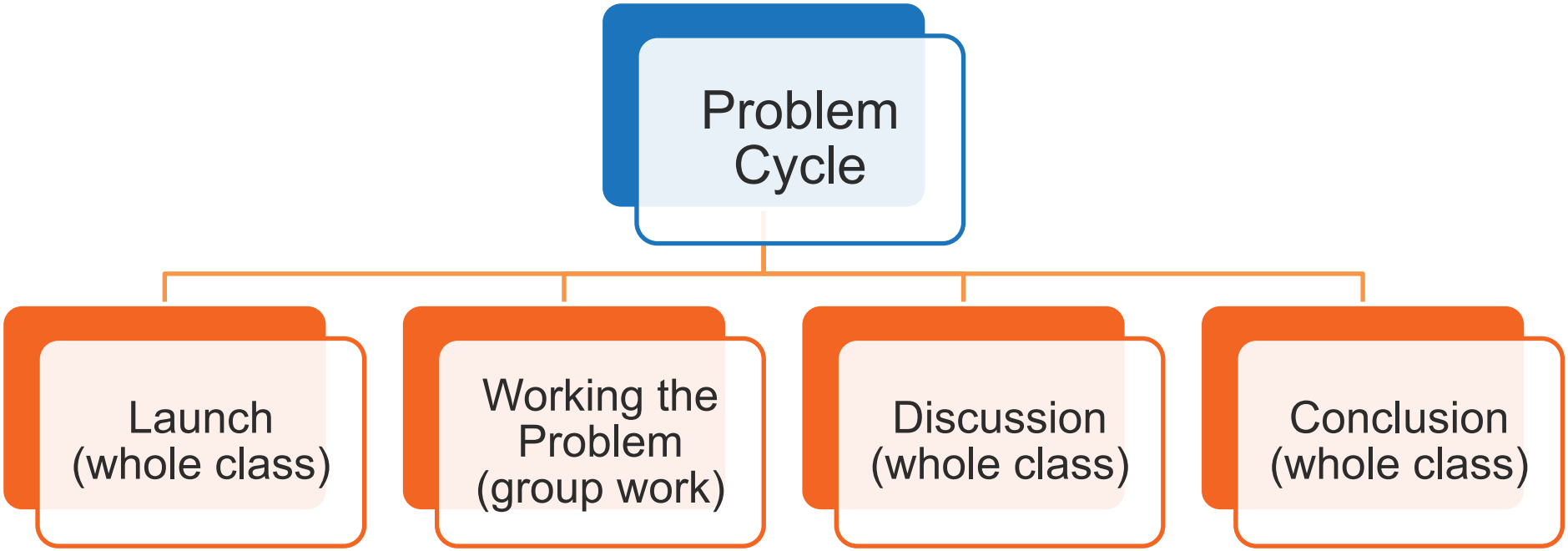
- To make explicit the relationships among and between mathematical facts, procedures, and concepts, as well as the contexts in which the mathematics is engaged
- To allow students to place the new idea in relation to what they already know thereby making more robust current understandings and laying the foundation for future learning
- To both provide students with opportunities to make those connections themselves, in interactions with their peers, and with the guidance of their instructor

How do we support explicit connections?

- Instructors can emphasize connections (between concepts, procedures, ideas, disciplinary knowledge) at multiple points in a lesson
  - Relating to prior concepts at start of lesson
  - Summarizing at the end of a lesson
  - During discussions of students' approaches
- Students should also have opportunities connections themselves
  - During small group discussions
  - During discussions of students' approaches

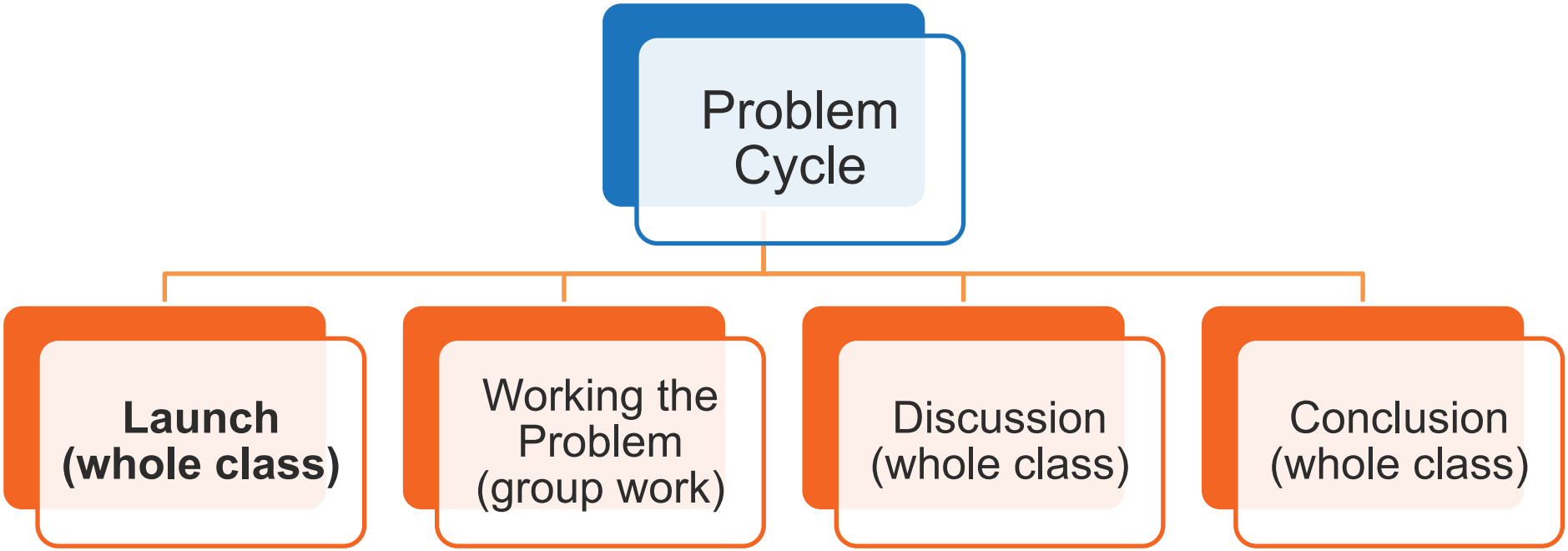
The Problem Cycle

An instructional routine that supports instructors to create opportunities for students to learn through productive struggle on rich problems and to engage in meaningful collaborative learning.

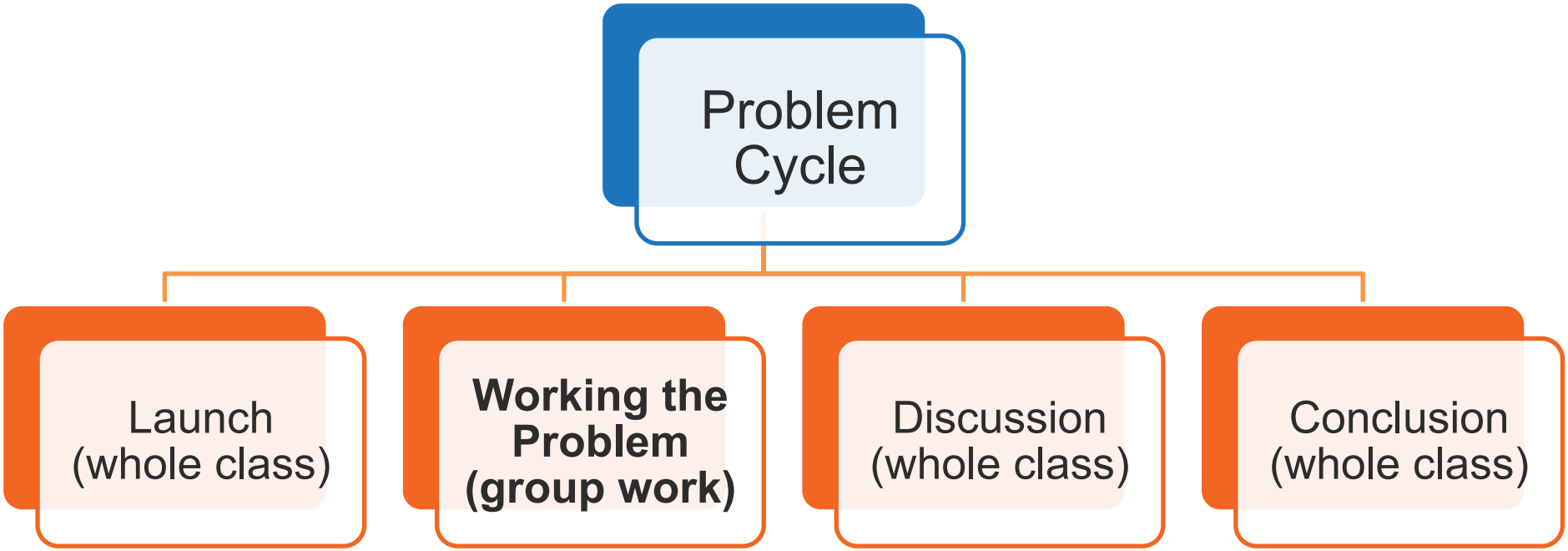


- Designed to support productive struggle
- Collaborative learning is critical component
- Makes student thinking/reasoning visible
- Builds key mathematical ideas from student thinking
- Leaves students with key takeaways from problem

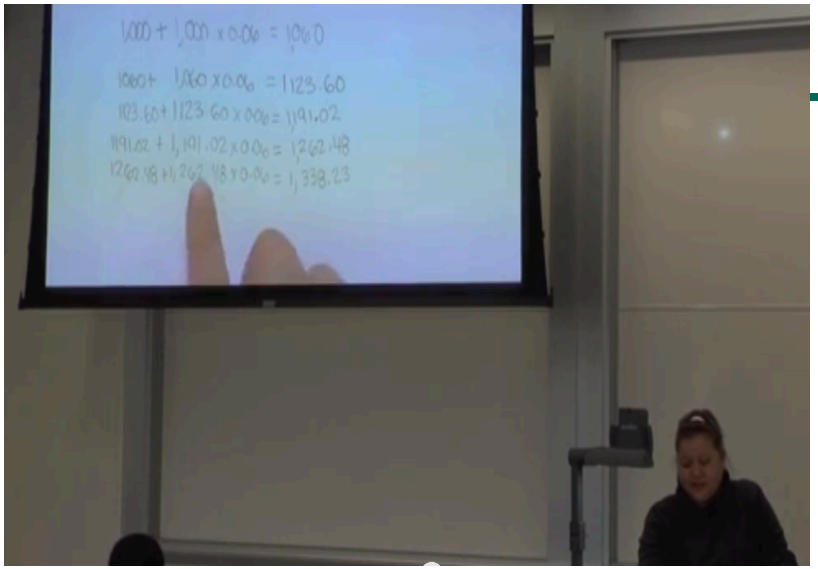

compounding interest



Problem Launch: The purpose of the launch is to prepare students for productive struggle -- to create a shared understanding of the problem to be worked on, make clear why solving it is important, and stimulate a variety of ways to think about the problem.



Working the Problem: The purpose of the working phase is to engage students in *productive struggle* with the problem and the concepts and to study students' ways of thinking to prepare for the discussion. The purpose of this phase is NOT to ensure that all students get the correct answers.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1060 + 1060 x 0.06 = 1123.60
1123.60 + 1123.60 x 0.06 = 1191.02
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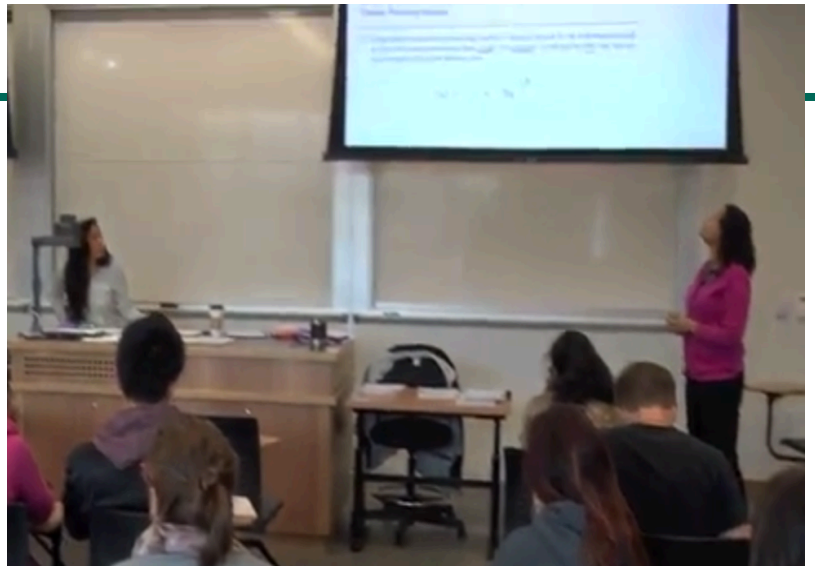
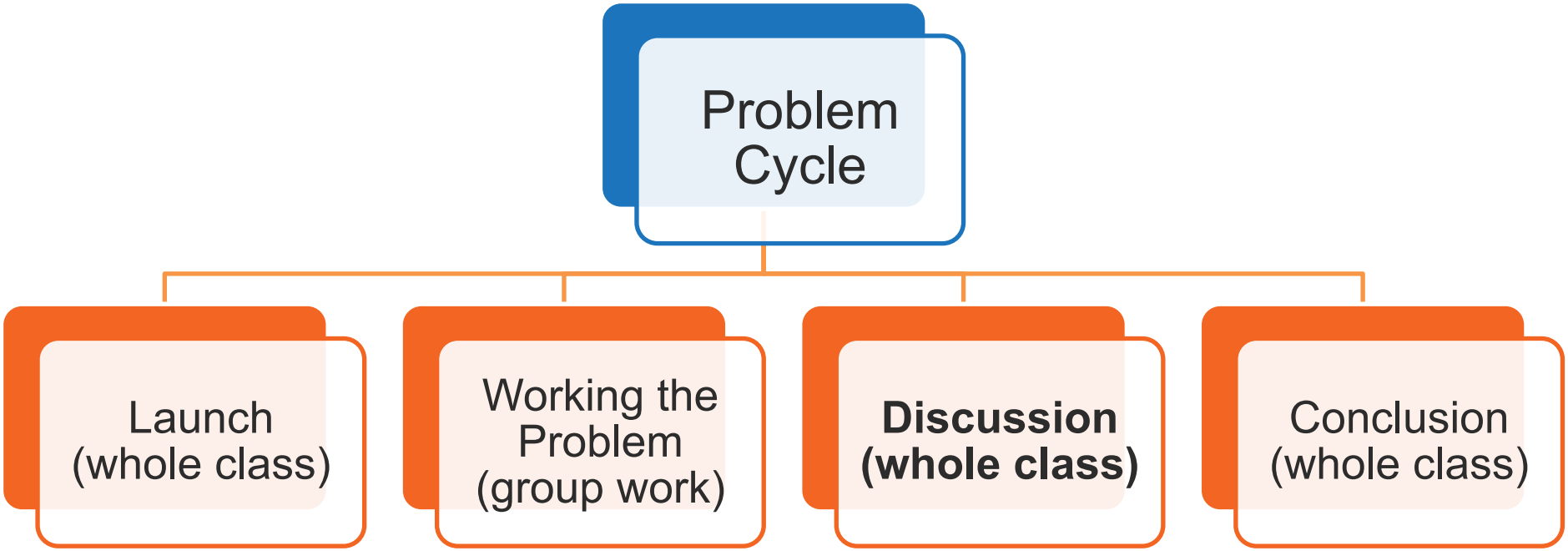
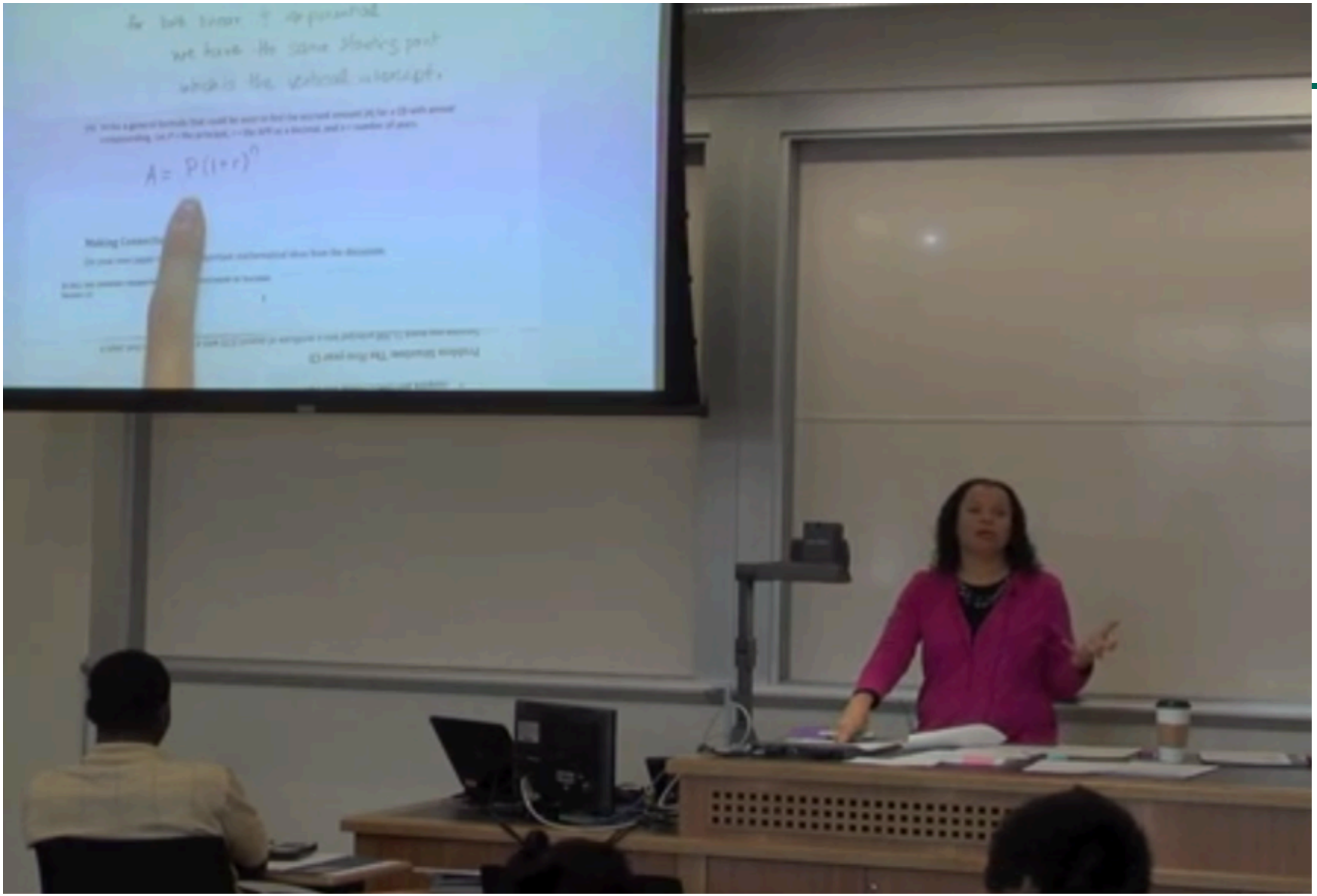




Discussing the Problem: The purpose of discussing the problem is to make public students' ways of thinking (correct and incorrect), encourage students to learn new ways of thinking by understanding each other, and *connect their thinking to the key concept(s)*.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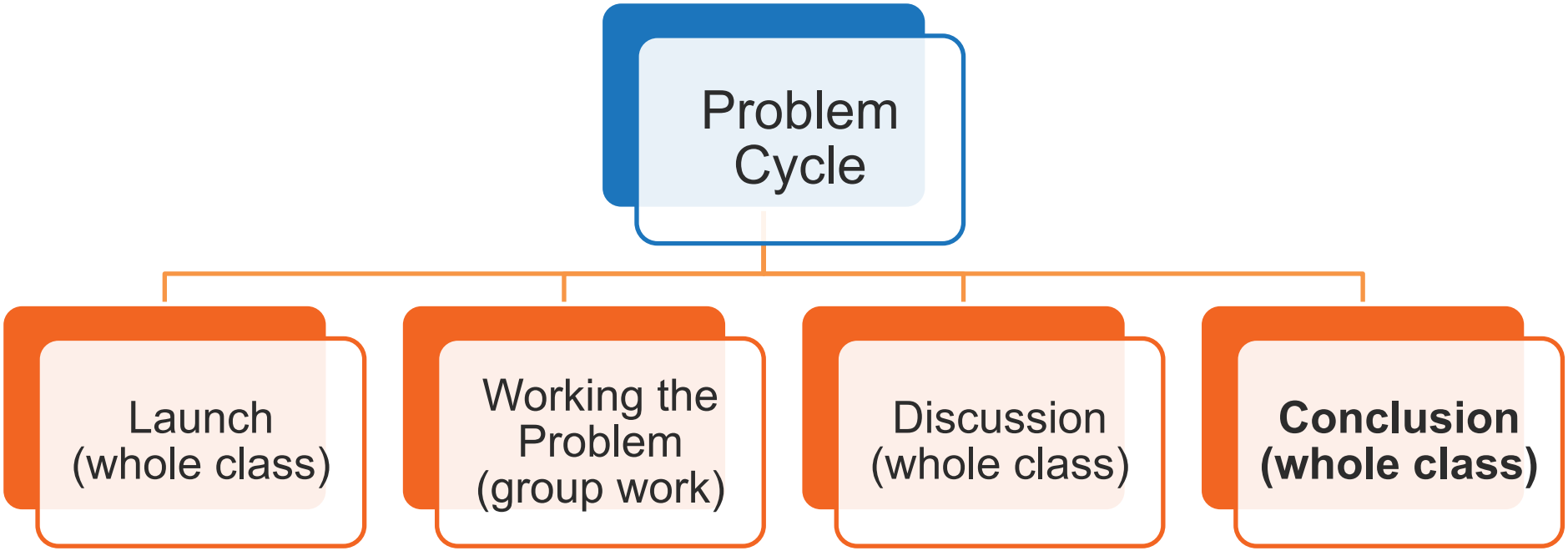

Conclusion: The purpose of the conclusion is to concisely highlight the key concepts drawn from students' thinking, express the concepts with appropriate notation and representations, *and explicitly connect the lesson concept(s)* with the course organizing concepts.

Using the Problem Cycle

- Can do one or a few PC's in a lesson
- Important to be explicit with students about norms and expectations
- Works best with tasks designed to promote productive struggle and are about core concepts (mathematical and disciplinary)

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