Instructor's Notes 3.1

Salary per Minute

Themes: Personal Finance, Medical Literacy

Main Math Topic	Main Quantitative Reasoning Context	Productive Persistence Focus	Level of Productive Struggle
Dimensional analysis	Several	Not applicable	Level 2

PREREQUISITE ASSUMPTIONS

Before beginning this lesson, students should

- be able to multiply two fractions.
- be able to divide two fractions.
- understand that a fraction can be simplified by "canceling" or dividing common factors in the numerator and denominator.
- understand that multiplying by 1 does not change a value.
- be familiar with basic units of measure for both length (feet, miles) and time (seconds, hours, minutes).

LEARNING GOALS

Students will understand that

- the units found in a solution may be used as a guide to the operations required in the problem—that is, factors are positioned so that the appropriate units cancel.
- units can add meaning to the numbers that result from calculations.

Students will be able to

- write a rate as a fraction.
- use a unit factor to simplify a rate.
- use dimensional analysis to help determine the factors in a series of operations to obtain an equivalent measure.

EXPLICIT CONNECTIONS

Canceling units is based on the same mathematical concept as canceling common

factors in the numerator and denominator of a fraction. This is actually a form of division and is based on the fact that anything (except 0) divided by itself is equal to 1, and that multiplying a number by 1 does not change the value of a number.

NOTES TO SELF

One thing I want to do during this lesson …

One thing I want to pay attention to in my students' thinking …

One connection or idea I want to remember …

SUGGESTED TIMELINE

Duration	Activity	Suggested Structure
10 minutes	Introduce the concept and methodology of dimensional analysis and lead students through Question 1	Class discussion
10 minutes	Let students work on Question 2	Small groups
4 minutes	Review Question 2 with class	Class discussion
8 minutes	Let class work on Question 3	Small groups
4 minutes	Review Question 3 with class	Class discussion
4 minutes	Briefly introduce the context of Question 4 to class	Class discussion
10 minutes	Questions 4-6 and Making Connections	Small groups, then class discussion

SPECIAL NOTES

In this lesson, rates are purposely written in different forms so that students will become familiar with the different forms. You may need to clarify that these forms are equivalent (e.g., miles per gallon, miles/gallon, mi/gal).

PROBLEM SITUATION: USING DIMENSIONAL ANALYSIS

Dimensional analysis is a method of setting up problems that involves converting between different units of measurement. It is also called *unit analysis* or *unit conversion*. Many professionals—including pharmacists, dieticians, lab technicians, and nurses—use dimensional analysis. It is also useful for everyday conversions in cooking, finances, and currency exchanges. Many people can do simple conversions without dimensional analysis; however, they will likely make mistakes on more complex problems.

The advantage of using dimensional analysis is that it is a way to check your calculations. While it is always important that you develop your own methods to solve problems, this is a time when you are encouraged to learn and use a specific method. Once you have learned dimensional analysis, you can decide when to use it and when to use other methods.

(1) (a) According to Toyota's website, a 2014 Prius can get an estimated 51 miles per gallon (mpg) in the city and 48 mpg on the highway.[1] How many miles will you be able to drive in the city if you have 4.5 gallons of gas?

Answer: 229.5 miles

(b) How many gallons of gas will you need to drive 3,450 miles?

Answer: 67.6 gallons

Note: Students might calculate the gallons of gas needed for highway driving (3,450 ÷ 48 = 71.875) or possibly calculate this with an average of city and highway mpg: 3,450 ÷ 49.5 = 69.697.

CONVERTING YOUR PAYCHECK

This is an example of how to use dimensional analysis to solve a problem.

Sample Question: Your paycheck for two weeks came out to $1,200. You work eight hours a day, five days a week. How much are you making per minute in cents? (**Hint:** You will need to use dimensional analysis to solve this problem.)

Answer: First, decide what you are looking for. Here you are looking for the rate of cents per minute, which can be written as:

$$\frac{¢}{\text{min}}$$

[1] "Prius 2014," *Toyota*, accessed July 14, 2014, http://www.toyota.com/prius/#!/Welcome.

Now, we start with a rate that we know has the same **numerator** (the part of a fraction that represents a count of the number of parts) as the rate we are looking for. Since cents converts to dollars, we have:

$$\frac{100¢}{\$1} \qquad \rightarrow \frac{¢}{\text{min}}$$

Next, we want to cancel the "$" (since this unit is not in the answer we are looking for). To cancel the "$" we can multiply by a unit ratio with "**$**" in the *numerator*. Since $1,200 is the amount we make in two weeks, we can multiply by that ratio:

$$\frac{100¢}{\$1} \times \frac{\$1200}{2 \text{ weeks}} \rightarrow$$

Now, we need to cancel the unit "weeks", so our next ratio must have "**weeks**" in the *numerator*:

$$\frac{100¢}{\$1} \times \frac{\$1200}{2 \text{ weeks}} \times \frac{1 \text{ week}}{5 \text{ days}} \rightarrow \frac{¢}{\text{min}}$$

Continuing this process, we finally get to the unit factor for "minute":

$$\frac{100¢}{\$1} \times \frac{\$1200}{2 \text{ weeks}} \times \frac{1 \text{ week}}{5 \text{ days}} \times \frac{1 \text{ day}}{8 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \rightarrow \frac{¢}{\text{min}}$$

(2) Is the resulting calculation reasonable?

Answers will vary. The resulting calculation is 25 cents per minute. Students could check the "reasonableness" of this by calculating how many dollars per hour this is. This is a familiar way to express salaries. Twenty-five cents per minute is 1,500 cents per hour, or $15 an hour.

Dimensional analysis is a mathematical tool or device for problems involving units and/or conversion. This is one realm of the course in which discovery learning principles are not easily applied. In other words, it is useful to "show" students the technique used for Questions 1 and 2. The technique is explicitly written out in the sample question above in part to insure that students have a clear example, as they would in a textbook, for how to apply dimensional analysis. You should lead the class through this example, instructing students on the technique. Different instructors have different strategies for using dimensional analysis; the strategy exemplified above embodies an algorithmic approach, wherein the desired units for the answer "leads off" the calculation, setting up a chain that ends by the cancellation of all units but the desired set.

Note: In Questions 3 and 4 students begin to practice the dimensional analysis technique outlined in the sample question (located above Question 2). Students may be expected to struggle with this technique, while others may pick it up quickly. Some students may insist that they can do it in a "common sense" way that doesn't involve dimensional analysis at all. Give some credence to these alternative methods, while noting that there will most likely come problems later for which dimensional analysis will be useful.

(3) Many states have banned texting while driving because it is dangerous, but many people do not think that texting for a few seconds is that harmful. Suppose you are driving 60 miles/hour and you take your eyes off the road for four seconds. How many feet (ft) will you travel in that time? **Hint:** Start with the unit you are looking for (ft). Then, create a chain of ratios, starting with one where "ft" is in the numerator that will cancel *all* other units. Remember that there are 5,280 ft in one mile.)

Answer: 352 feet

(4) In Module 2 we examined population densities and used these to calculate projected populations. The population density of Tokyo is 6,038 people per square kilometer (km).[2] Use dimensional analysis to calculate how many people would live in the nation of Japan, which comprises an area of approximately 378,000 square km[3], if the entire nation was as dense as the city of Tokyo.

Answer: 2,282,364,000 people

Note: Preface Questions 5 and 6 by noting that dimensional analysis is an essential skill in dosage calculations, which are necessary for any nurse to master. You might briefly review the conversions between metric scale measurements. Here, students will find it more difficult to calculate the answer without using dimensional analysis.

(5) Nurses are often required to calculate dosages. That is, they must check the order that a doctor has given for the administration of a drug and decide whether the dosage is correct. To calculate correctly they must convert between different metric units. For example, 1,000 milligrams (mg) = 1 gram (g); and 1,000 micrograms (mcg) = 1 mg. Also, 1 gram = 1 milliliter (mL).

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[2] "Population of Tokyo," *Tokyo Metropolitan Government*, accessed July 14, 2014, http://www.metro.tokyo.jp/ENGLISH/PROFILE/history03.htm.

[3] "East & Southeast Asia: Japan," *The World Factbook, Central Intelligence Agency*, last modified June 20, 2014, https://www.cia.gov/library/publications/the-world-factbook/geos/ja.html.

Suppose a doctor has ordered a dose of 0.1 gram of a medication. The drug comes in a solution concentration of 200 mg per milliliters. How many milliliters of this solution is required?

Answer: 0.5 mL

(6) Now, calculate how many milliliters you would need to administer 500 mg from a dosage concentration of 1 g per 3 mL.

Answer: 1.5 mL

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

MAKING CONNECTIONS: MAIN IDEAS TO HIGHLIGHT

Main Idea: *Canceling* units is based on the same mathematical concept as canceling common factors in the numerator and denominator of a fraction. This is actually a form of division and is based on the fact that anything (except 0) divided by itself is equal to 1 and that multiplying a number by 1 does not change the value of a number.

Note: You might consider if you want to use the term *canceling* as it tends to obscure this understanding—it is important to get students to explain the process mathematically. Students will often use the term *canceling*, but it is more correct to refer to the operation as division.

Facilitation Prompts

- What does it mean to "cancel" units? Why is it allowed?
- How does this relate to simplifying or multiplying fractions?

FURTHER APPLICATIONS

(1) Do an Internet search for "dimensional analysis" or "unit analysis". Find at least one site that provides examples of how to make conversions using this technique.

(a) Record the site name and URL address.

(b) Copy one example of a conversion using dimensional analysis (as shown on the site).

Below is an example problem from "Fun with Dimensional Analysis," from Alysion.org:

> *At the pizza party you and two friends decide to go to Mexico City from El Paso, TX where y'all live. You volunteer your car if everyone chips in for gas. Someone asks how much the gas will cost per person on a round trip. Your first step is to call your smarter brother to see if he'll figure it out for you. Naturally he's too busy to bother, but he does tell you that it is 2015 km to Mexico City, there's 11 cents to the peso, and gas costs 5.8 pesos per liter in Mexico. You know your car gets 21 miles to the gallon, but we still don't have a clue as to how much the trip is going to cost (in dollars) each person in gas ($/person).*[4]

[4] "Fun with Dimensional Analysis," *Alysion.org*, accessed July 24, 2014, http://www.alysion.org/dimensional/fun.htm.