Learning Opportunities for Classrooms in the Carnegie Math Pathways[*]

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Context

The Carnegie Foundation for the Advancement of Teaching launched the Carnegie Math Pathways (formerly Community College Pathways) in September 2009 to aggressively improve the failure rates of developmental math students in community colleges across the country. To achieve this goal, Carnegie engaged practitioners, researchers, designers, developers, and institutional leaders to design and implement two distinctive pathways— Statway® and Quantway®—to challenge and fundamentally change the character of developmental mathematics. These pathways target students who are at grave risk of failure in mathematics courses at the community college level—students who have weak K-12 preparation, face language challenges, or fundamentally believe that they are destined to not do well in the subject. Both Statway and Quantway seek to reverse a pernicious and disheartening cycle of failure for too many students by employing materials and teaching approaches that fundamentally put them on a pathway of success. This essay explores three learning opportunities that form the foundation for the curricular design and pedagogical approach of these curricular materials.

Introduction to the Learning Opportunities

Knowledge about mathematics teaching and learning, as well as more general principles about learning, suggests that certain learning experiences are important for deep learning. At the broadest level and across all subjects, the National Research Council in *How People Learn* (2005) determined that there are three basic principles of learning: 1) new understandings are constructed on a foundation of existing or prior understandings; 2) the brain forms cognitive schema or networks that are important to emphasize in the learning process; and 3) the ability to self-monitor or possess skills of meta cognition enhance learning.

The Carnegie Math Pathways address the essence of these fundamental principles in a number of ways. Foremost among them are the three "learning opportunities" embedded in the instructional design—productive struggle, explicit connections to concepts, and deliberate practice. These three learning opportunities are interconnected and not necessarily exclusive of each other. They form a foundation for the lessons within these new innovative pathways for developmental math students. Below we discuss each of these learning elements.

[*] This essay was originally written by K. Merseth. It has been modified slightly to reflect changes in naming of the Pathways and of some curricular components since the original writing of this essay.

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What is productive struggle and what does it look like in community college classrooms?

What is productive struggle?

Productive struggle in Pathways lessons seeks to engage student thinking in challenging learning opportunities with the intention to engage students in thinking about important mathematics concepts. Productive struggle causes students to explore as they develop strategies and their own thinking about the use of mathematics to investigate a problem situation or question. The ultimate goal of productive struggle is to encourage students to make meaning of mathematical content for themselves.

Students who are productively struggling are engaged and inquiring, and repeatedly making guesses and judgments about how to use mathematics to approach the given situation. The task they face requires thinking and the students have both the time and encouragement within the classroom culture to engage with the problem.

The focus of the productive struggle is on the mathematical learning goals embedded in the problem or situation—it's not about guessing what the teacher wants to hear or about finding a particular answer. It is about the process of thinking, making sense, and persevering in the face of not knowing exactly how to proceed or whether a particular approach will work. Exploring, investigating one or multiple approaches, and articulating a chain of reasoning behind the approaches also characterize productive struggle.

Another internal dimension of productive struggle is self-awareness in a problem-solving situation. It can involve gaining perspective of where you are in the course of solving a problem and assessing what approaches might be helpful in proceeding (or even getting unstuck) to address a math problem.

A classroom with students engaged in productive struggle might be quiet at first, but then it may explode into lively discussion within several small groups. During the small group discussions, students might try out ideas with each other, critique those ideas, and together build/create different ideas and understandings that help move the understanding of the problem forward. A group engaged in productive struggle is not given a procedure for how to do the work and a problem capable of supporting productive struggle likely has multiple approaches or solutions. The instructor circulates around the room listening to students and helping them to articulate their reasoning by asking questions and probing for clarification of the students' thinking. The instructor also keeps the students from heading too far down an unproductive path by carefully asking questions that might cause the students to reconsider their approach or by challenging them with discrepant evidence that prompts such a reconsideration. The nature of the discussion about the problem is more about approaches and concepts. The nature of the discussion about the problem would likely be more about approaches and concepts than on procedures. While some might say that the instructor provides hints, the instructor usually doesn't explicitly offer answers or directions, but rather offers comments designed to keep student thinking proceeding productively.

What will the students be doing during productive struggle?

The task given to the students is one that students have an inkling that they can accomplish though the exact approach may not be immediately obvious. There will be some trial and error and even some silence in the classroom. During silence, the students might be contemplating an approach to the problem, perhaps writing some notes on the paper, jotting down a few ideas. (Silence, of course, also can indicate confusion and despair. Before intervening, instructors may wish to see whether the reflective silence leads to action. Body language can be helpful in deciding if a student is engaged or not. For example, a student leaning into the group, making eye contact with others, writing on the paper or otherwise participating is likely engaged, even if silent throughout).

Frequently students join in small groups to discuss their thinking and ideas. During the discussions, there will be sharing of ideas about strategies, justifying answers, students questioning each other, and looking at each other's papers. The students' questions will be likely related to mathematical concepts or ideas, not computation, and this will suggest that the students are grappling with key concepts (and not trivial or extraneous ones). Productive struggle in groups also is characterized by students influencing each other's approaches to a problem, causing ultimately a reformulation of original thinking and approaches when communicating with a partner.

What is the faculty member doing during productive struggle?

Faculty are not inactive while students are engaging in productive thinking. Indeed, faculty are usually quite active, circulating around the room checking in with individuals and groups, asking probing and provocative questions, and assisting groups that appear to be bogged down and feeling stumped. Sometimes faculty intervene directly with students, usually on an individual basis or with a small group. Directive intervention and struggle are not incompatible, but rather it is the *quality and nature of the intervention* that differs from more traditional teaching.

How and when does a faculty member decide to intervene? One indicator is body language. Are students avoiding eye contact or only looking at their own papers? Is the group uncharacteristically silent? Does the group seem disengaged, bored? Are pencils down? To determine non-productivity, the faculty member needs to listen to and observe the groups carefully. Other indicators of non-productive struggle might include students rehashing the same ideas over and over or continually throwing out ideas without actually pursuing them. Another warning sign is if the faculty member sees that the group is diverging wildly from the mathematical goal of the lesson. The key for the instructor is to watch and listen carefully in order to discern what the students are thinking.

Obviously when to intervene is a judgment call. Some researchers have stressed that there are no clear and unambiguous indicators of what to do when, just as there are no clear teaching moves that always work in every classroom situation or with every student. Knowledge, perspective, and experience are key to what, in the end, is a professional judgment. Knowing your students well is perhaps the most important criterion informing faculty intervention. One individual has likened intervention decisions to making popcorn in a microwave oven. You listen and watch and then act when you think just enough popcorn has popped. Too soon and you waste good popcorn; wait too long and, well, burned popcorn just doesn’t smell as good.

What are indicators of non-productive struggle?

Indicators of non-productive struggle vary from classroom to classroom, student to student. However, tell-tale signs include student frustration, helplessness, exasperation, inactivity, and outright anger. After an initial burst of activity, students may put their pencils down, and they may look confused. Or students may persist in a seemingly neurotic fashion, trying the same thing over and over again. Some students may belligerently declare “You know the answer! Why don’t you just tell us?”

Other indicators include when the discourse shifts so that the teacher is doing all of the talking and the students are silent. Or the teacher asks only fill-in-the-blank or yes-no questions (sometimes intended more to check engagement in the task than to discern understanding).

How does a faculty member prepare to stimulate productive struggle?

There are several important tasks for faculty to undertake when preparing to teach a class in which struggle is productive and not debilitating. The first is to prepare students for the experience. Many students are accustomed to having the faculty member tell them exactly how to accomplish a task, outlining the required procedures and asking students to practice this procedure many times with similar problems. The intention of Pathway materials is not to tell students how to do something but rather to encourage students to think for themselves. The faculty member’s role, to use an old cliché, is to be the “guide on the side, not the sage on the stage.” Students may not be accustomed to faculty behaving in this way and may express frustration unless the faculty member has taken the time to describe how Pathways materials and teaching approaches are different from other math classes the students may have experienced. Thus, preparing students for productive struggle by *emphasizing the process rather than the result, the way of thinking rather than an exclusive focus on the answer* is warranted. Faculty members might also discuss why this approach is beneficial to actually learning the mathematics and retaining what is learned

Faculty members will also want to establish clear classrooms norms in which the expression of ideas, concepts, and hypotheses are encouraged and honored. All comments and observations about the mathematics are encouraged and no question is ever a ‘dumb’ question. This may mean that the classroom is occasionally noisy with students conferring with others. Further, some students may interpret group problem solving as ‘cheating’ so the creation of a classroom culture in which collaboration is encouraged, indeed required, is paramount It would be wise to also set norms for the quality of and role in discourse (not blurting out answers, rolling of eyes, groans, etc.). Clearly, reminders about patience and an honoring of the inquiry process are fundamental to the classroom culture. At base, the faculty member wants to create a welcoming, safe environment in which to the free exploration of thinking and ideas occurs.

Second, the faculty member will choose problems or questions that ideally are compelling to students. The context should motivate students so they genuinely would be motivated to search for an answer or explore multiple answers. The task itself, maybe somewhat complex and novel, and slightly beyond immediate comprehension by the students. The idea is that the particular task will stretch or provoke student thinking. The Pathway material seeks to accomplish these goals.

Third, the faculty member should be well aware of the mathematical learning goals for the lesson. Because there are no iron-clad faculty teaching moves that work in every situation and with every student, it is more important for the instructor to be clear about the *mathematical concepts and explicit connections within the material as well as the instructional goals for the students*. Further faculty should prepare by considering what kind of thinking might the task provoke in students. What are the potential misconceptions—places where students may go wrong in their thinking? It is a good idea for faculty to prepare (prior to the class) some possible hints, or leading questions that do not give the answer or approach, but rather guide student thinking in a more productive direction. Research on teaching has found that anticipating misconceptions and asking specific questions about the misconceptions can significantly aid learning.

Where does productive struggle come into play in the instructional system of the Pathways?

Productive struggle frequently may appear in the initiating lessons of a particular module. It is here that lesson designers choose to put forth idea and concepts that may be slightly beyond the immediate grasp of students. Of course, productive struggle may appear in other parts of the lessons as well as in the online homework materials.

What are explicit connections and when do they occur in Pathway classrooms?

What are explicit connections?

Within the Pathway materials the idea of explicit connections refers to the linkages or relationships among and between mathematical and/or statistical facts, procedures, and concepts. Explicit connections generally reference math ideas and concepts and may be about context as well. Connections may be drawn by students or faculty, but most often are presented and reinforced by faculty.

Explicit connections can be very simple. For example, when a mathematics teacher has introduced the formula for finding the distance between two points on a coordinate plane, she might say: “Can anyone see a connection between the distance formula and the Pythagorean Theorem?” This could be followed by drawing a triangle to connect two points on a plane, and then having students explain the relationship, using gestures to point out connections between the two ideas. Another example, from statistics, might include the following: When students are being asked to calculate a p value to determine how likely it is that an observed mean came from a given population, the faculty member might ask, “What does this probability mean?” and then follow up that question with, “Which distributions are we using as our point of reference?” In this way, the faculty member would remind students of the distinction between a population distribution and a sampling distribution, and also of the fact that both are theoretical distributions (as opposed to empirical).

Explicit connections also might appear when the same mathematical concept is presented in different mathematical forms—algebraically, graphically, or numerically. Another example would be percentages. Percentages are used in different ways and sometimes with different names: as a relative measure, as a relative measure of change or difference, as a probability, as indices (such as the CPI), but these are all connected by the foundational meaning of percent. The explicit connection is for students to see the common concept instead of viewing each of these applications as a discrete and unrelated topic. Another example is a geometric transformation of a figure in a plane which can also be expressed as an algebraic transformation. While they look very different, they are expressing the same mathematical concept of a transformation.

Ideally, an explicit connection helps students place the idea into existing cognitive schema they possess. Doing this enables the learner to gain a deeper understanding of a concept and ‘where it fits.’ Making connections explicit also is part of a process of stepping back from mathematical problems and thinking about mathematics as a field. In other words, explicit connections help provide insights about the structure of mathematics and how it all fits together. Students may, for instance, turn solutions and theorems over and over so they can be seen from many perspectives and—importantly, because they are connected to something the learner already understands—become more memorable.

Making explicit connections helps reinforce earlier learning and builds a strong foundation for future understanding. Essentially this means what students will do today with good scaffolding, they can do by themselves tomorrow. Explicit connections enable students to routinely ask themselves questions that constitute mature mathematical learning habits: how is this like something I already know, how is this related to other ideas or techniques that I have studied before, and what kinds of problems can I solve with this?

Explicit connections also elevate the work of the mathematics student from driving for a specific externally validated answer such as that found in an answer key to a focus on making sense of and appropriately using mathematical concepts. In this way, the “correctness of a solution” is not based on external validation, but rather results from the student’s personal sense making. In such a situation, students know they are correct without needing to consider someone else’s work or to check an answer sheet. Thus, when connections are made by students, their work becomes a measure of self-validation and empowering. This experience results in the affirmative statement “I get it!” Ultimately, with explicit connections, students come to see mathematics as coherent, rational, and logical.

A final aspect of explicit connections is that they make more evident fundamental mathematical and statistical principles that are often “disguised” by changes in terminology, context, notation, and procedure. For example, ratios and percentages are based on the same fundamental principle of rational numbers, but because they are used in different contexts where they look different (in their commonly used forms), many students experience “learning percent” as unrelated to “learning ratios” or “learning proportions.”

When, where, and by whom are explicit connections made?

Faculty may emphasize explicit connections at any point during the class. Some may introduce a concept by relating to prior concepts in earlier classes or they may make reference to connections relevant to the day’s lesson at the end of a class during summary comments. The more that students can connect the knowledge to something that they already know the easier it will be for them to remember. Explicit connections would certainly occur in lesson wrap-ups.

Students also may make connections when they are struggling with a particular problem. For example, a student might suddenly say, “This reminds me of…” Or “Does this work the way that other problem worked?” Or it can occur as an “ah-ha” moment when the student makes a link between different math concepts and says “I get it now. It’s like…” Sometimes connections are made in the middle of a class discussion. Whenever or wherever the linkage is made, an explicit connection helps students place the idea into a cognitive schema that the student already possesses which enables the student to remember it more easily the next time the concept appears.

Not all explicit connections are made within the classroom. Given well-designed activities, students may make explicit connections when using the online homework materials.

How can faculty facilitate and prepare for explicit connections?

In general, the instructor should prepare by identifying what the possible connections are before teaching the lesson. Given this information, the faculty member can have strategies prepared to bring these connections to life. Sometimes these connections are made explicit in the materials but always they rely on and benefit from a clear articulation, either by students or faculty.

Clearly, the essential ingredients for faculty to make important connections include a deep understanding of mathematical and statistical content and structure, an overview of the course, an understanding of the course objectives, and a comprehensive grasp of the learning goals for the lesson.

How are explicit connections handled in the instructional systems of the Pathways?

Faculty may find references to explicit connections in the Instructor’s Notes that accompany each lesson. Here, authors may make connections to other materials explicit and offer suggestions for how faculty can share these connections in the classroom.

What is deliberate practice and what does it look like in Pathway classrooms?

What is deliberate practice?

Deliberate practice consists of a set of tasks for students that are created to overcome gaps in understanding, apply what has been learned, and/or deepen fluency with key concepts. Activities that qualify as deliberate practice represent a strategic progression of exercises that is purposefully designed to improve performance and strengthen cognitive understanding. The key of deliberate practice is that it addresses the question: How should we select and implement problems over time to build greater conceptual understanding? Deliberate practice is not characterized by rote drill or repetition—nor is it the same problem structure repeated with the numbers changed. A slightly different interpretation of deliberate practice is being able to apply what is learned to new situations and to build learning through practice with scenarios that differ slightly and purposefully from those experienced previously. Performance of deliberately designed activities should be carefully monitored to provide feedback to students and important information to instructors regarding ways to improve learning.

Deliberate practice requires students to think about the approach they are using to solve the problems, why they are choosing that particular approach, and how the approach must be adapted for appropriate use in a new context or problem. It is different than doing 30 repetitions of the same type of problem. The design of deliberate practice very strategic and seeks to cause students to focus on why they are using a specific approach and less on learning a procedure.

Where does deliberate practice occur?

Deliberate practice may occur within a set of class activities or as part of out-of-class exercises. In either case, deliberate practice references the overarching architecture of the set of problems and tasks. The strategic design may evolve within a specific problem set or over a period of several days or weeks.

How can faculty enhance deliberate practice?

Faculty can consciously and strategically employ in-class and out-of-class activities in a deliberate way if they keep in mind the learning objective of the lesson and its related exercises. By thinking through a progression of skills and concepts, faculty can help engineer progress for students toward meeting the larger learning goal. Each exercise in a problem set, as well as the collective whole, should have a specific cognitive purpose, whether it is to approach the concept in a new and different way, to explore the application of the concept in a new context, or to promote facility in the use of important concepts, processes, or techniques. Often these require students to stretch their learning and to apply it in a new way. A collection of exercises should lead to something or somewhere; mindless repetition has been shown to be a remarkably inefficient (and certainly a highly demotivating) approach to gaining conceptual understanding.

Where does deliberate practice come into play in the instructional system of the Pathways?

Deliberate practice is seen most frequently in a series of tasks within lessons and in the collection of homework problems that accompany lessons.