## Lesson 4.2: More on Linear Modeling

LESSON 4.2

### SPECIFIC OBJECTIVES

Students will understand that

- linear models are appropriate when there is a situation with a constant increase/decrease.
- slope is the rate of change.
- the rate of change (slope) has units in context.
- different representations of a linear model can be used interchangeably.

Students will be able to

- label units on variables used in a linear model.
- make a linear model when given data or information in context.
- make a graphical representation of a linear model.
- make a table of values based on a linear relationship.
- identify and interpret the vertical and horizontal intercepts in context.

### PROBLEM SITUATION 1: MAXIMIZING BUSINESS REVENUE

A home business manufactures custom made necklaces. They conducted market research and found that 200 necklaces are sold each week if the price per necklace is $12.00. If the price is lowered to $10, then 300 necklaces are sold.

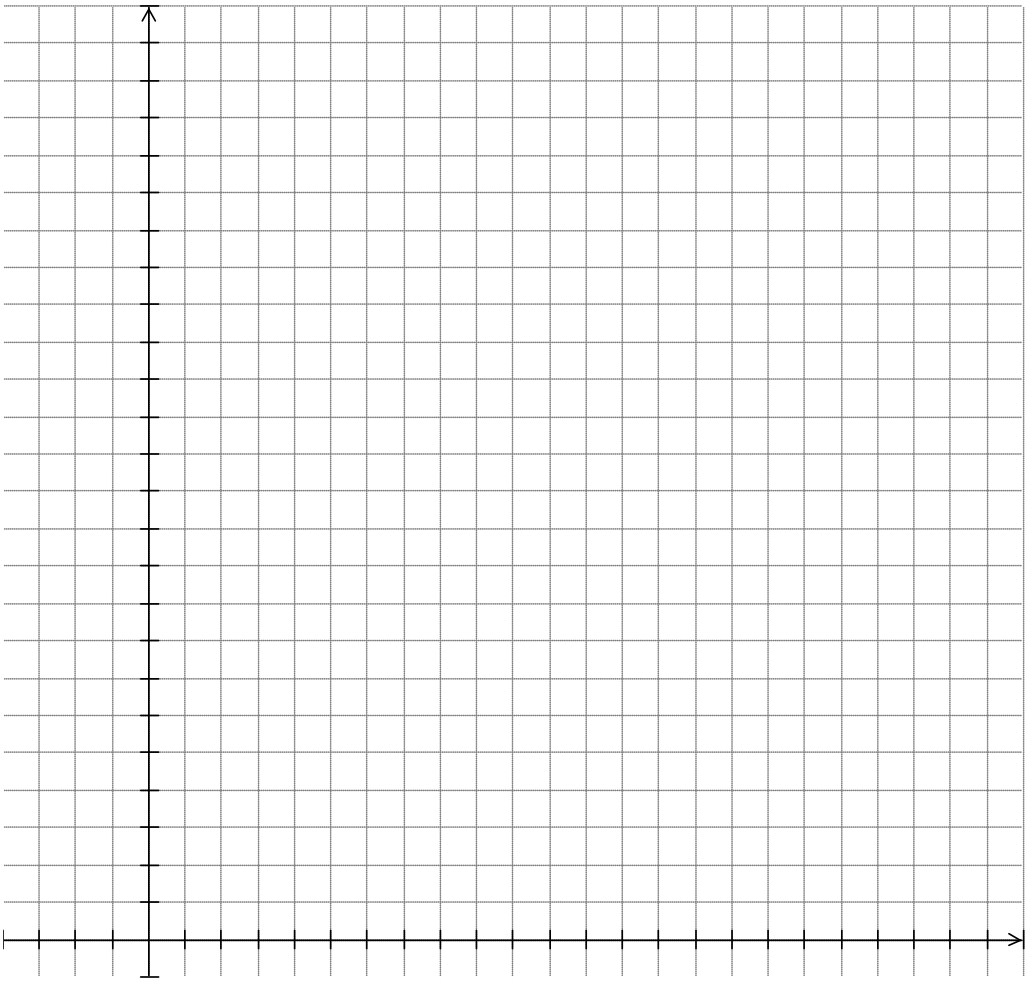
(1) Which is the better price from a business standpoint?

The business owner wonders if another price level would produce even more revenue. She decides to create a linear model that will help her to predict the revenue at different price levels.

She defines the following variables:

N = the number of necklaces sold
p = price of the necklace

(2) Let p be the variable on the horizontal axis and let N be the variable on the vertical axis. Design a graph, below, with appropriate scales for the two variables, and plot the above data obtained from market research.



(3) According to the graph, what is the predicted number of necklaces sold for a price of $8.00?

(4) What would be the expected revenue at this price?

(5) Decide what price would result in the most weekly revenue.

(6) Find the slope of this linear model. Include units in your answer. Explain the meaning of the slope in the context of this problem.

(7) Find the vertical intercept of the graph, and use this, with the answer to the previous problem, to construct the linear model relating N and p.

## PROBLEM SITUATION 2: BREAK-EVEN POINT

In business, the money a company spends is called *cost*, and the money the business takes in is called *revenue*. Generally the business needs to sell enough goods to make up for the costs. The number of goods needed to make up for the costs is called the *break-even point*.

Consider a small business selling hand-designed dresses made in a small loft building. The rent on the loft is $1,200 per month. There are two employees, each of whom are paid $20 an hour for 8-hour days, 5 days a week. Each dress costs $40 in materials to make.

(1) To mathematically analyze this business's *costs*, a model can be created. Define the variables that will go into the model for *cost*.

(2) Create a mathematical model for the (approximate) business costs.

(3) What kind of mathematical model is this?

(4) It is decided that the dresses should sell for $80 each. Create a mathematical model for this business's revenue. Be sure to define all variables.

(5) Now use the models created to find the break-even point, the number of units that must be sold to break even.

## MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

## FURTHER APPLICATIONS

(1) Suppose, in the dress business discussed above, that you realize this break-even point is too large. You are worried your customer base is not big enough to support such a number of sales. However, you do think that some customers will be willing to pay more for the dresses. What would be the price per dress that would enable you to break even at 50 dresses sold? (Hint: One way to determine this break-even point is to use the same equation that you solved in (11), but some of the values will be different)

(2) "In a *New York Times* article (April 23, 2010), Dr. David Felson, a rheumatologist and arthritis prevention specialist at the Boston University School of Medicine, said, referring to osteoarthritis (O.A.):

> "A woman's risk for developing O.A. is linearly related to her weight."

Explain in your own words what Dr. Felson means by "linearly related."