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Strength in Numbers

Collaborative Learning in Secondary Mathematics

Ilana Seidel Horn



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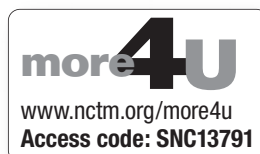
Collaborative Learning in Secondary Mathematics

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Providing Access to Meaningful Mathematics: Groupworthy Tasks

So far, I have introduced several ideas that aim to shift our thinking about what teaching secondary mathematics means. In chapter 1, I argue that, to encompass all that children learn in school, we need to move away from a notion of teaching as *effective presentation of ideas* toward a view of teaching as *designing effective learning environments*. In chapter 2, I discussed equity and offered four interpretive principles teachers can use to make learning environments more equitable. In chapter 3, I introduced *status* and described how it influences mathematical learning and how teachers can cultivate equal-status interactions through valuing different kinds of mathematical smartness.

Two things are worth noting here. First, although the vignettes in earlier chapters portray group work, the application of these main ideas extends beyond small-group settings. This is because group work as a learning environment is embedded in other aspects of instruction. Second, while I have hinted at an underlying conception of content, I have not yet fully explained what the mathematics itself might look like in an equitable classroom that uses collaborative learning effectively.

Just as I proposed in earlier chapters different ways of looking at the classroom, teaching, and students, I develop here a perspective on mathematics compatible with equitable collaborative learning. The shift can be characterized as one that goes from *ready-made mathematics* to *mathematics in the making* (the latter based on studies of scientists at work, described in Latour [1997]). That is, instead of having children learn what has already been figured out, we teach children mathematical content through sense making so that they learn not only *how* to do mathematics but also *why* it works. In this chapter, I will describe how this broader view of mathematics supports equitable group work.

Broadening Mathematics

Rigor and accessibility are often viewed as competing goals in mathematics teaching. The concept may seem paradoxical at first, but increasing rigor actually can support greater access. The resolution of the paradox comes from the particular strategy for broadening content. Some may bristle at the idea of broadening mathematics, assuming that doing so entails watering down the subject. Instead, by expanding school mathematics to make it more closely resemble the work of mathematicians, we deepen the integrity of the content and simultaneously make it more accessible.

The shift toward mathematics in the making is a move toward *mathematical proficiency*. This form of mathematical competence has five strands (Kilpatrick, Swafford, and Findell 2001, p. 5):

1. *Conceptual understanding*—comprehension of mathematical concepts, operations, and relations
2. *Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
3. *Strategic competence*—ability to formulate, represent, and solve mathematical problems
4. *Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
5. *Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

When students can demonstrate these forms of mathematical thinking, they do more than just learn mathematics. They can do mathematics. This vision of content aligns well with the rich mathematical learning environments that support equitable collaborative learning.

Researchers Magdalene Lampert and Deborah Ball conducted some of the first teaching experiments seeking to bring mathematics in the making into elementary school classrooms. Unlike earlier attempts to bring authentic mathematical thinking into the classroom, Lampert and Ball paid close attention not only to the structure of the content but also to how children learned mathematical ideas. Their rich classroom records, which include videotapes of daily lessons, lesson plans, teacher journals, and student work, have yielded enduring images of what teaching mathematical thinking might look like (Lampert and Ball 1998).

Important mathematical practices in their classrooms included reasoning, justification, building definitions and representations, and reconciling seemingly different approaches to problems. These mathematical thinking practices are the key to bringing deeper content while supporting greater access to sense making. Figure 4.1 illustrates some of the mathematical habits of mind that can enrich classroom learning.

Some Key Ideas about Mathematical Learning

Lampert and Ball's work was pivotal in helping teachers envision what might be possible in mathematics classrooms. Thanks to their work and the work of other mathematics education researchers, we know a lot more about how people learn mathematics since the first NCTM *Standards* were written in the late 1980s. *Principles and Standards for School Mathematics* reflects much of this research (NCTM 2000; Kilpatrick, Martin, and Shifter 2003).

Here are some key ideas from this research:

- *To use knowledge flexibly, students need to understand what they are learning.* Recitation and memorization generally allow students to develop a limited mathematical competence. They may be able to produce an answer when given a similar question to one that they have seen, but they are often stumped when they need to use their knowledge in new situations. Also, when students learn ideas superficially, they tend not to retain them.
- *New understandings build off prior understandings.* Students do not come to mathematics classes as blank slates. They have a set of experiences, intuitions, and ideas about number and space. Effective teaching requires that these prior understandings be engaged in the classroom. The metaphor of a *scaffold* is often used to describe how teachers might start with students' conceptions of a topic and build toward conventional mathematical understandings. To be valuable, scaffolds must engage students' understandings, not simply wallpaper over them.

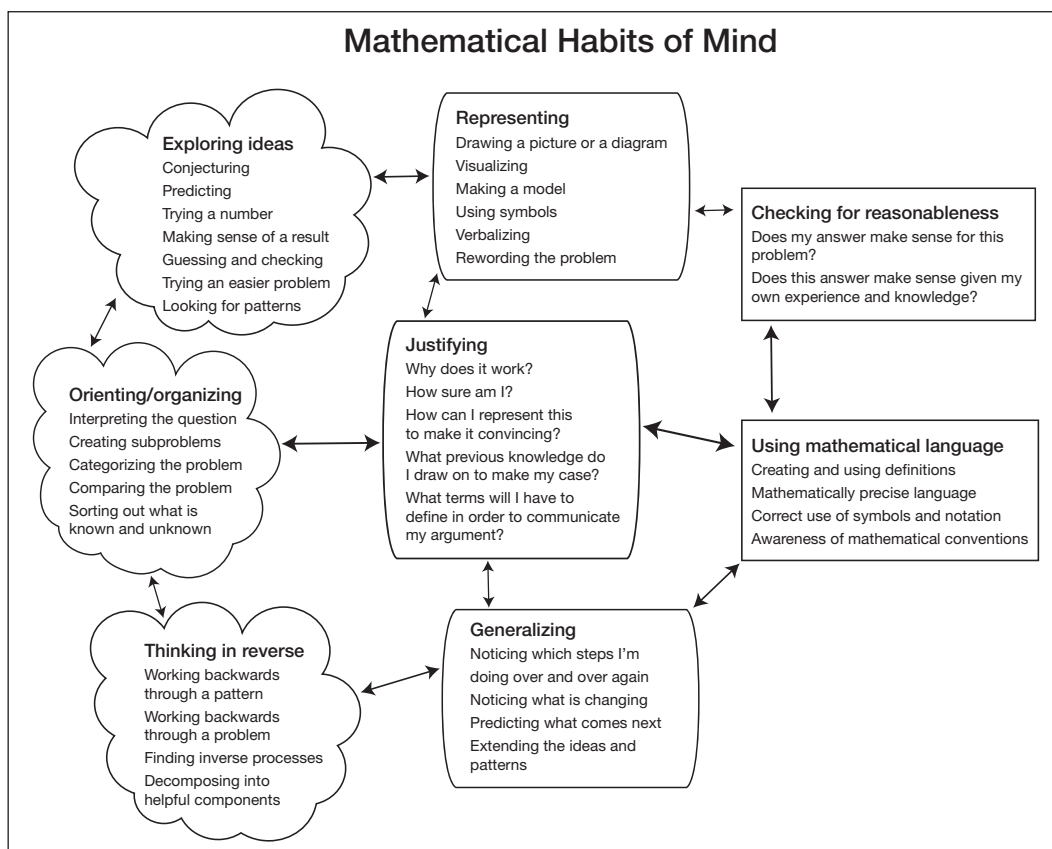


Fig. 4.1. Some mathematical habits of mind, along with the mathematical activities or questions that support their development

- Mathematical practices such as argumentation and justification support student understanding.* Argumentation, justification, and generalization are thinking practices that mathematicians engage in. They also support students' understandings of content by helping them learn not just the *how* of mathematics but also the *why*. Argumentation and justification support the scaffolding of student thinking because asking students to justify their thinking requires engagement of their prior understandings.
- Students need to be encouraged to see themselves as a source of mathematical knowledge.* One great feature of mathematics as a subject is that it makes sense. If its underlying principles are understood, others can be deduced. If students lose sight of the underlying logic, they take an approach of memorizing from a textbook. Although this approach may help them pass tests or do problems that are closely related to the ones they practice on, they typically become paralyzed when they encounter a slightly different problem. They also forget what they have learned too quickly. If students know how to reason mathematically, they can often think their way through difficult problems and better retain information. This is where mathematical proficiency comes in.
- How students learn influences cognition, motivation, affect, and sense of self.* We live in an era that focuses mainly on the cognitive outcomes of learning. Classroom teachers must contend with the interrelated issues of motivation, affect, and self-concept. A learning environment perspective on teaching helps make sense of these other dimensions of

instruction, because the contexts of learning influence students' learning of content. For example, they influence students' persistence on difficult problems and in the subject over time, as well as their feelings about mathematics, including contributing to or detracting from math anxiety. As we discussed in chapter 3, the status dimension of classroom interactions can profoundly affect how students think of themselves as mathematics learners.

New Ideas, New Dilemmas

Most educators would not argue against the idea of teaching for understanding. Intuitively, people usually realize that they learn better and have more flexible knowledge when they have a deep understanding of an idea. To teach with this goal in mind certainly requires a departure from the traditional mathematics classrooms described in chapter 1. Teaching mathematics for understanding poses new challenges.

For instance, how do we build off the prior knowledge of thirty people in one classroom? Getting a handle on *one* student's understandings of a single curricular topic can be challenging enough. Also, how is a lone teacher to do this for every student in every class for every topic, when secondary teachers teach five or six sections a day? Likewise, when we dig into student understandings, we might stumble upon gaping holes in their knowledge base, which leads to a new dilemma. Do we follow the students or follow the curriculum?

Although complex instruction cannot entirely resolve these dilemmas, effective small-group learning can help address some of these tensions for teachers. To achieve efficacy, we first must address some common assumptions about mathematical learning.

Mathematics Is a Group of Connected Ideas

To productively direct students' sense making, teachers need to shift away from a primarily hierarchical view of mathematics to a connected view. Although we tend to think of mathematics as progressing in a sequence, with students needing to wholly master prerequisite skills before they can learn new ones, this is often not the case. To be clear: mathematics certainly has a logical deductive structure, yet nobody would propose teaching out of the *Principia Mathematica*. (The *Principia Mathematica* is a three-volume work on the foundations of mathematics, written by Alfred North Whitehead and Bertrand Russell and published in the early twentieth century. It sought to derive all mathematical truths from a set of axioms by using logical deduction.) Bringing this point closer to secondary content: understanding algebra before having gained fluency in multiplication facts might be difficult, but there is no reason why students cannot develop the concept of variable and inverse operations to solve equations *while* they work on their computational fluency in multiplication. Although teachers need to respond to gaps in student knowledge, the missing pieces do not necessarily require a complete reteaching of older topics.

Mathematics Is Not Strictly Hierarchical

The opposite of a *connected view* of mathematics is a *hierarchical view*. In this perspective, mathematics is a sequence of topics that necessarily build off one another. In traditional secondary mathematics curricula, this sequence culminates in the study of calculus. A connected view of mathematics, in contrast, emphasizes the discipline's big ideas and what have been referred to as *habits of mind* (Cuoco, Goldenberg, and Mark 1996). In the algebra example, helping students develop a concept of variable and of inverse operations emphasizes understanding strategies for equation solving. Big ideas are generalizable and appear throughout the study of mathematics. Inverse operations simply point to the mathematical habit of mind that Mark Driscoll (1999) refers to as *doing-undoing*: anything we do in mathematics, we seek to undo. This premise holds true for arithmetic operations as well as for functions in calculus and linear transformations in groups. The

principle extends beyond the specific topic of study, so it is generative for students' future mathematical learning.

The increase in mathematical rigor comes from bringing school mathematics closer in line with mathematics itself. The broader notion of mathematics itself provides an important shift for the success of complex instruction. By incorporating *more* mathematical skills in our teaching, we give students more opportunities to demonstrate competence. These different mathematical skills are the source of the multiple abilities that allow us to address status in our classroom.

Turning Some Pet Ideas about Mathematics Teaching on Their Heads

Before we construct tasks that will support collaborative learning, we must first challenge a few pet ideas about mathematics teaching. Certain mathematical teaching practices come from the hierarchical view of the subject. When we shift to a connected view, we select and organize tasks differently. Doing so is particularly important for successful group work.

Start with Challenging Stuff, Not Easy Stuff

Classroom mathematics tasks tend to be organized so that a simple example is presented first, followed by similar problems that gradually get harder. The problem set may or may not culminate in a challenge problem.

What's wrong with this approach? On the surface, it makes sense. If learners are anxious, we want to build their confidence by allowing them the opportunity to be successful. In collaborative learning, though, this approach is a disaster for several reasons. By presenting a problem that maps easily onto an example, teachers inadvertently encourage students who see patterns quickly to take over the task without involving their groupmates. Because good pattern seers tend to succeed in school mathematics, this structure supports their ongoing dominance and reinforces existing status problems. Also, leaving the meaty problems for the end may deprive many students from ever getting to the substantive content. In fact, some students believe challenge problems are optional or "only for the smart kids." Well-chosen challenge problems serve as a good starting point for group work *because* they help students get directly to the heart of mathematical issues. Finally, by starting with challenging problems, teachers send the message that all students can engage with difficult content, especially when students have each other as resources. Repeated experience with difficult problems supports students' development of the kinds of strategies that strengthen their mathematical reasoning.

In sum, starting with the easy stuff contributes to inequitable teaching. Putting challenging content at the end of assignments limits who has opportunities to engage with these problems and, in doing so, perpetuates the opportunity gaps that limit student learning. In addition, making challenging problems essentially optional means that many students are not pushed to learn mathematics more deeply.

Effective Group Learning Allows All Learners to Help Each Other

I often hear teachers talk about collaborative learning as offering opportunities for "fast learners" to slow down and teach, while struggling learners get in-class tutoring from their peers.

This usual way of thinking about collaborative learning contributes to status problems. As I said in chapter 3, students quickly size up any underlying design of your groupings. They will recognize if they are being put in the "smart one" or "minority student" slot, and they will often act accordingly. Productive mathematical conversations weigh arguments on the basis of mathematical validity, not on who is speaking. The very act of putting students in slots assigns them a role linked to their presumed competence, socially endowing them with different levels of authority regardless of what they say. Likewise, if we truly create a multiple-ability classroom centered on

problems that require a range of mathematical skills, then the added dimensions of mathematical competence should scramble any hierarchical ranking of student ability.

Using Groupworthy Mathematical Tasks

Complex instruction requires a thoughtful design of mathematical activities. Not all classroom work is well suited for collaborative learning. To distinguish between tasks that are and are not, teachers have developed the idea of *groupworthy tasks*.

To illustrate this concept, I will take a problem that has these properties and explain how you might use it to make it groupworthy. Take a moment to look at the task (fig. 4.2) and think through how you might do it. Then think about what students might do.

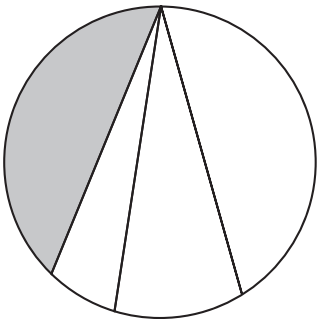
Fractions or Not?

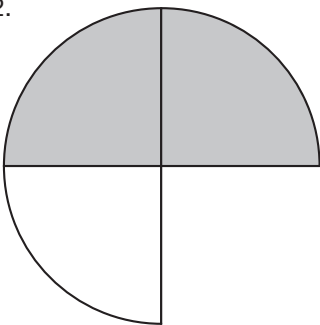
Looking at the following pictures. With your group, decide if the picture is a fraction or not.

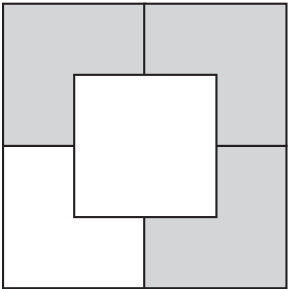
Then:

- If you think the picture is a fraction, decide what is the **part** and what is the **whole**. Then write the fraction.
- If you think the picture is *not* a fraction, explain why not.

Make sure everybody in your group can answer the teacher's questions about how you came up with your answers!

1.

2.

3.

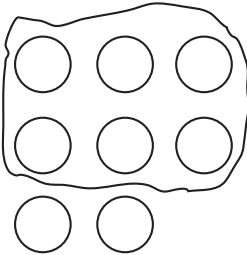
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Fig. 4.2. The *Fractions or Not?* activity

Groupworthy tasks have six common features (Lotan 2003). To be groupworthy, a task needs to do the following:

- *Focus on central mathematical concepts or ideas.* Once we shift our thinking about mathematics from a sequence of topics to a network of ideas, we can identify which ones are important to teach. (For more resources on processes that help identify what is important to teach, see Wiggins and McTighe [2005].) In general, they are the topics that students will encounter again and again, that support long-term understanding of the content, and that illuminate important concepts. Our goal is to help students develop important insights about these ideas so that they can use them flexibly in a variety of contexts, in our class and in the future.

As all secondary-level mathematics teachers know, computations with fractions are the downfall of many U.S. citizens. The mere presence of a fraction can petrify some students. This activity helps students revisit the definition of fractions and asks them to apply it to nonstandard diagrams. The teacher who shared this task with me had a goal of working on conceptual fraction problems in her class to ensure that everybody had a good grasp of what fractions were, providing a conceptual resource for them to use in other nonstandard problem contexts.

- *Require some interpretation.* Groupworthy tasks incorporate multiple intellectual abilities. To support equal-status interactions in small-group contexts, teachers need to disrupt classroom status hierarchies. Doing that is possible only if students have intellectually meaningful ways to contribute different perspectives to a task. The *Fractions or Not?* task requires multiple abilities. To work on this activity successfully, students need to use the definition of a fraction and apply it to nonstandard contexts. Diagram 2, in particular, pushes students to reconsider a familiar shape—a 90° sector of a circle, which usually represents $\frac{1}{4}$ —and understand why in this situation it is actually $\frac{1}{3}$. To see this, students need to be able to reinterpret visual cues and make arguments for their interpretation of the different diagrams.
- *Provide multiple ways of being competent in problem solving.* Sometimes when teachers are first using complex instruction, they think that long or multistep problems qualify as groupworthy. This second criterion of groupworthiness is important because it sets the stage for students' discussion. Students at a broad range of achievement levels should have a way into the problem. Diagram 3, in particular, is fruitful for student discussion because it can be looked at in two different ways. Either the white square in the middle can be seen as *part* of the whole or it can be viewed as empty space, leaving the other four L-shaped parts as the whole. I have seen students passionately debate the nature of this diagram, carefully invoking the definition of fractions in the process. Creating an open-ended task like this that forces student interpretations allows discussion of different approaches, creating a context for students' justification of their thinking and the reconciliation of diverse conceptions.
- *Be done in a group, which bolsters students' interdependence.* A well-known issue with collaborative learning comes when one student takes over the cognitive work of a task while the other students sit back and socialize. To eliminate this free-rider effect, teachers need to ensure that tasks actually *require* the input of multiple students to work effectively. (I will talk more about fostering interdependence in chapter 5.)

A single student working on this problem would not see all the different possible ways to interpret the diagrams, and therefore the underlying concepts would not necessarily surface. This problem works *better* in groups than it would alone. Also, the nonstandard diagrams force a considered interpretation, slowing students down and pushing them to reassess their automatic responses.

- *Be designed in a way that provides individual and group accountability.* Related to the concern about students' simply using others' thinking without doing any of their own, accountability systems need to be in place that require all students to contribute. I will get into more detail about accountability strategies in chapter 5, but I will present one example here to illustrate. Classroom routines are one way to communicate norms and expectations, including ones involving accountability. In many complex-instruction classrooms, teachers answer *group questions* only during collaborative learning time. If a group of students calls the teacher over, the teacher will ask, "Is this a group question?" If not, the teacher says, "Ask each other first, and then if you still don't know, I will come back." If the students answer affirmatively, the teacher will then call on *any* student to state the question. The group question routine increases mutual accountability and individual accountability. Students cannot get their individual questions answered without first discussing them with their group. If the teacher's help is still needed, each student must be prepared to articulate the question and answer the teacher's follow-up questions about it.
- *Have clear evaluation criteria.* Clear evaluation criteria are essential to any good assessment. Articulating to students what you are looking for makes your expectations visible to students. It supports the goal of autonomy because it allows students to know whether they have fulfilled the requirements of the task. It also provides consistency across groups of students. Evaluation criteria should be written in language that students can understand, and should inform them about what constitutes exemplary work. For example, "An outstanding poster will have the problem statement, your strategy, a solution statement, and a clear justifications for your reasoning. Students from other groups should be able to walk up to your poster and understand what you did." For a smaller task such as *Fractions or Not?*, teachers might have simpler goals, such as, "Anyone in your group should be able to explain your answers to my questions when I check your work. Make sure you have *reasons* for your answers." Clear evaluation criteria support more focused and higher quality student discussions, as well as better final products (Cohen and Lotan 2003).

"We know that students often walk into our math classes filled with fear but also with hope. Math has caused them to feel stupid in the past, but they are hoping that this year, with this teacher, will be different. More than anything else, the tools of [complex instruction] have helped me face this enormous responsibility. For example, an emphasis on groupworthy tasks means that I am asking students to work on harder mathematics—harder because I am asking them to justify, use multiple representations, generalize, connect, apply, and reverse processes. This lets me catch more students being smart and challenge every student to work on getting smarter."

—Carlos Cabana, Complex Instruction Educator and Mathematics Teacher

Other Notes about Task Design

Finding a groupworthy problem poses the first challenge to organizing content. Usually, problems need to be reformatted to make them suitable to group work. One dilemma of group work is that, to air their conceptions, students in groups need some autonomy from the teacher. But teachers cannot simply turn the conversations over to students without significant forethought. Teachers need to ensure that student conversations center on important ideas in the curriculum. One tool for managing this delegation of authority is the *task card* that teachers distribute to groups. A task card supports group autonomy by giving students the problem to have at their tables, not just at

the board. It differs from a worksheet by giving guidance for the activity, but it is not meant for students to write on or turn in. Giving two cards to a group of four or five students forces students to share resources, another way to foster interdependence. Some teachers put their task cards in plastic sleeves to reuse them for multiple class sessions.

Because we want students to work during their collaborative time with as little teacher intervention as possible, thought must go into some of the details of the task card design. Here are some things to think about in designing a task card:

- *Layout.* Ideally, the task should be clearly laid out, with a set of simple directions and diagrams. Task cards should use an appropriate font size and include a clear sequence of directions, diagrams, probing questions, and evaluation criteria. Take care not to proceduralize the task, however. Leaving some ambiguity gives students something to talk about.
- *Language.* A mathematical task should not become a reading comprehension exercise. Problems need to be simply stated. Most word processing programs have an easy way to check for the reading level of any text you produce. Aim for two grade levels below your students' grade. Doing so may require finding words with fewer syllables and breaking text into shorter sentences. Be mindful of English language learners, and note any mathematical language or problem contexts that might be unfamiliar.
- *Representations.* Diagrams, number lines, and graphs are some representations that are central to mathematics. Not only do they potentially push students' thinking, but they might also serve as a resource for students who are good visual thinkers. Also, English language learners might be able to make sense of a problem using diagrams or manipulatives more successfully than they could a standard word problem.
- *Evaluation criteria.* On a task card, communicate to students what you are looking for, either on their written work or in their conversations. Doing so lets students know that they are all individually accountable for the group's thinking and emphasizes the teacher's interest in the justification for the responses.

By allowing students to work together on groupworthy tasks, teachers give students opportunities to develop academic language. From chapter 1, recall that the IRE (initiation–response–evaluation) instructional format allows only one turn of talk from one student to respond to a known-answer question. This format does not give students opportunities to develop verbal fluency in academic language or cultivate mathematical habits of mind. When concepts are developed in small-group settings with richly represented problems, students can first engage with ideas by using local language. For instance, students might first observe an increasing linear function by noting, “Look! This is going up,” or, even further from academic language but with just as much intuition, they might slant their arm at an angle and say, “It goes like this.” Their descriptions can get progressively more technical as students first interact with each other. One might ask, “How do you know it's going up?” and hear in response, “The x and y values are increasing.” Later, with their teachers or the whole class, they can learn that this regular increase can be attributed to *slope*.

This layering in of representations, observations, and academic language supports all students' concept development, as the abstract terms emerge from observations they made in their own interactions. Mathematics educator Judit Moschkovich (1999) argues that developing ideas through representations is particularly important for English language learners, stating that diagrams, gestures, and graphs do more than provide extralinguistic cues; rather, they become a focus of meaning making for students.

Summary

In an equitable mathematics classroom, content needs to be reconceptualized in two important ways. First, mathematical knowledge includes more than just knowing *how* to do problems; it includes *how* and *why* certain approaches work and make sense. This conception not only moves school mathematics closer to the work of mathematicians, making it more rigorous, but also makes it more accessible. Justifications for why mathematics works give students an important entry point in mathematical thinking and help them understand and retain what they have learned.

This connection to student learning helps reconcile the seeming contradiction between making mathematics more rigorous and simultaneously accessible. In fact, in the past thirty years, substantial research has shown characteristics of robust mathematical thinking. If we want students to use their knowledge flexibly, they need to understand what they are learning. To help them understand, we must engage their prior conceptions. As has been suggested, mathematical thinking practices such as argumentation and justification support such understanding while encouraging students to view themselves as a source of mathematical knowledge. Finally, we are increasingly aware that students learn more than just content in school; they learn who they are in relationship to academic knowledge. Learning environments shape students' motivation, affect, and identities in ways that go beyond the traditional ideas about learning content.

Widely held assumptions about mathematics teaching do not hold true in a collaborative learning context. First, not only do teachers not need to start off with easy problems; starting with easy problems actually works against complex instruction. Challenging problems require greater interdependence than easy ones and send students the message that they can do difficult mathematics. Second, group work should not be viewed as an opportunity for smart students to teach struggling students. In a truly rich mathematical environment that supports multiple abilities, all learners help each other see problems from different perspectives.

For these ideas to be effective, students must be given mathematical problems that are group-worthy. That is, problems should (1) focus on central mathematical ideas, (2) be open-ended and incorporate multiple intellectual abilities, (3) have some aspect that is open to interpretation and offer multiple entry points into their solution, (4) work best when done in a group to bolster student interdependence, and (5) be designed and implemented to ensure both individual and group accountability.

Working with group-worthy problems can support the development of academic language, giving students vital opportunities to use mathematical terms and forms of talk while discussing ideas with their peers. This approach may particularly benefit English language learners, because the concepts can emerge from experiences with representations that can be accessed without heavy language demands. You can layer in increasingly complex academic language as students make observations about a problem together.

In this chapter, we focused on what an expanded view of mathematics that would incorporate mathematical thinking practices might look like and how those support students' engagement with challenging mathematical content. The example of the *Fractions or Not?* task illustrated some of those ideas in a familiar context. I talked about the group question routine as an important part of implementing the task in a group-worthy manner (fig. 4.3).

In the next chapter, I will elaborate on creating individual and group accountability systems in collaborative mathematical learning.

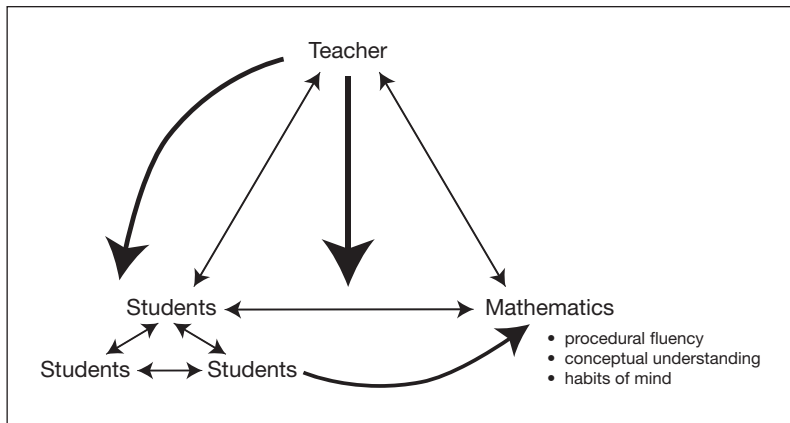


Fig. 4.3. Mathematics is broadened in the use of groupworthy tasks.



Ilana Horn is a professor of Mathematics Education at Vanderbilt University's Peabody College. Her work focuses on making rigorous mathematics accessible to a wide range of learners. Her research has two strands. The first considers what it

means to teach secondary mathematics equitably and ambitiously. The second investigates how mathematics teachers learn this form of teaching, examining the role of colleagues and contexts in teachers' sense making.

Students who work together, succeed together. Isn't that every teacher's goal?

Written by a seasoned teacher, researcher, and teacher educator with over two decades of teaching experience, the goal of this book is to support teachers in developing tools for effective group work in their secondary mathematics classrooms. The book outlines ways to choose tasks, help students adjust to new ways of approaching schoolwork, and discusses the types of status problems that can impede the most earnest attempts at collaborative learning. This practical, useful book, introduces tested tools and concepts for creating equitable collaborative learning environments that supports all students, and develops confidence in their mathematical ability.

- Use group work effectively to create a learning environment in the secondary mathematics classroom
- Learn how students experience learning mathematics in collaborative settings
- Develop tasks, concepts, strategies, and tools that create successful group work and reach students of all abilities

This incisive, informative, well-researched, and practical book describes ways in which teachers can use collaborative learning to create secondary mathematics classrooms in which all students have an equal opportunity to learn. It deserves a wide audience—including teachers, teacher educators, administrators, and policy makers.

—**James A. Banks**

*Kerry and Linda Killinger Endowed Chair in Diversity Studies
and Director of the Center for Multicultural Education*

Strength in Numbers addresses two crucial problems... The first is... how to engage all classroom learners in meaningful mathematics. The second is how to structure teaching so that teachers play a major role in improving the practice. Horn has done a beautiful job of grounding the relationship between these two challenges in real stories of teaching and teacher learning.

—**Magdalene Lampert**

*Professor of Education and Coordinator of the
Learning in, from, and for Teaching Practice Project
University of Michigan*

Horn offers practical principles, strategies, and insights that come straight from working in public, urban high schools. Mathematics teachers committed to group work, equitable participation, and building vibrant classroom communities should start here.

—**Carlos Cabana**

*High School Mathematics Teacher
and Complex Instruction Educator*

Inside, read more of what professionals think...