

Main Math Topic	Main Quantitative	Productive	Level of
	Reasoning Context	Persistence Focus	Productive Struggle
Measures of central tendency	Information Technology Text Messaging Data	Not applicable	Level 3

Prerequisite Assumptions

Before beginning this lesson, students should be able to

- perform basic operations using quantities such as integers, fractions, or decimals with the aid of technology.
- find the mean, median, and mode of a set of numeric data.

Specific Objectives

Students will understand that

- each statistic—the mean, median, and mode—is a different summary of numerical data.
- conclusions derived from statistical summaries are subject to error.
- they can use the measures of central tendency to make decisions.

Students will be able to

- make good decisions using information about data.
- interpret the mean, median, or mode in terms of the context of the problem.
- match data sets with appropriate statistics.

Specific Language and Literacy Objectives

Students will be able to

- read and comprehend the problem situation
- complete annotation and summarization with quantitative and information technology information from problem situation.
- demonstrate understanding of mathematics through complete and correct written answers to problems.
- demonstrate ability to describe, analyze, and synthesize information using measures of central tendency to looks at patterns of data.
- use appropriate quantitative and information technology vocabulary to discuss mathematics in this lesson.

Explicit Connections

- It is important to ask questions about quantitative information.
- Additive change maintains absolute differences. Multiplicative change does not.

Notes to Self

One thing I want to do during this lesson ...

One thing I want to pay attention to in my students' thinking ...

One connection or idea I want to remember ...

Suggested Timeline

Duration	Activity	Suggested Structure
5 minutes	Questions 1- Problem Situation and Annota- tion	Individual & Class Discussion
5 minutes	Question 2	Small Group & Class Discus- sion
10 minutes	Question 3 and 4	Small Group, Class Discussion, and Individual Writing
9 minutes	Question 5	Individual & Group
12 minutes	Question 6,7, and 8	Small Group
8 minutes	Question 9	Small Group
	Making Connections	
	Further Applications	

Special Notes

Today's lesson is specifically contextualized for information technology. In this lesson, students will be asked to think deeply about the important use of understanding measures of central tendency, with an emphasis on when to use median and mean. In PNL 2.6, students used a graph to answer questions about the mean and median of sale prices of new homes sold in the U.S. In the OCE 2.7, students will continue practicing mean and median in different scenarios. This lesson shows how measures of central tendency can be used to gain a deeper understanding of information technology issues. This lesson focuses on using mean and median to better understand trends and patterns about information technology.

Language & Literacy Support for the Lesson: This lesson places an important emphasis on writing in mathematics, especially when using median and mean to analyze data. You may want to review the Writing Principle with students prior to beginning the lesson. Discuss with students the importance of writing in complete sentences, and writing complete and precise answers. This includes using both quantitative and contextual information in their answers.

Included in this lesson are two approaches to supporting students' understanding of the mathematics lesson – *annotation of text* and *summarization*. Text annotation involves identifying main ideas and supporting evidence, key vocabulary words, or quantitative information. You can use annotation to ensure that students closely read the problem situation. Students who read and annotate text are better able to understand the content, and are better able to participate in classroom discussions. Summarization is used to support students' comprehension of the problem situation.

It is also important to check to ensure that students understand vocabulary words, the problem situations, and mathematical questions as you progress through the lesson as an additional language and literacy support for students.

Timing. This lesson will require the entire class period. You may want to monitor student discussion time to ensure lesson completion.

[Student Handout]

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Specific Language and Literacy Objectives

Students will be able to

• read and comprehend the problem situation .

- complete annotation and summarization with quantitative and information technology information from the problem situation.
- demonstrate understanding of mathematics through writing complete and correct responses to mathematics problems.
- demonstrate ability to describe, analyze, and synthesize information using measures of central tendency to looks at patterns of data.
- use appropriate quantitative and information technology vocabulary to discuss mathematics in this lesson.

Note: Begin the annotation activity by explaining the role of annotation in learning mathematics. Explain that the annotation helps students pay more attention to important information in the text. Next, point out the annotation symbols. Explain which symbols they will be using to annotate the text. Be sure to point out that 'quantitative information' is usually numbers. Also explain, to students, that annotating the problem situation will help them better answer the summarization (writing questions). Next, guide student use, on the overhead, annotating one or two sentences. Review the results with the students. Then ask students to complete the annotation on their own.

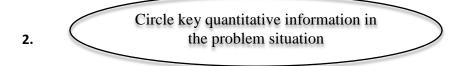
Have students read the problem situation individually. This will help them build their reading comprehension.

[Student Handout]

For this lesson, you will learn a skill called annotation. Annotation is a reading comprehension tool that helps readers read, and understand what they read. To annotate, you will underline the **key concepts** in the problem situation and circle the **key quantitative information.** Use the annotation symbols in Figure 1. Your instructor will annotate the first sentence or two with you, as a whole class. Then you will be asked to annotate the remainder of the problem situation on your own as your read.

Figure 1. Annotation Symbols

1. Underline Key Concepts in the Problem Situation



Problem Situation: How Do Students Text?

You are the Director of Information Technology at White Oak Community College (WOCC). Recently, students and faculty have been complaining to you about the online campus platform. It can be difficult to communicate about assignments and school events on the platform. Most students at WOCC do not have laptops or computers at home. Instead they use smartphones or tablets to do school work. Students also complain that it takes instructors too long to answer messages about homework assignments. Instructors complain that students do not check their messages often enough, or do not receive information about course work.

The Committee for Virtual Learning (the Committee) proposes to install a mass text messaging system on campus. You believe that this is a good idea. This system, called EducText, would allow students and faculty to text about assignments, course work, and anything school related for free. It would help instructors provide quick responses to students, and send text messages to the entire class at one time. EducText would allow the school to communicate time-sensitive information, using text messages, to students and staff about urgent safety issues, school closings, and weather alerts.

Given the significant costs of implementing EducText, the Committee has decided that implementation is only cost-effective if most students send or receive text messages at least 20 times a day. You assume that most students are active texters, but you must conduct research to understand how students at WOCC use text messaging in their daily lives. This will help you determine if EducText is worth the cost. You conduct this research during the 2014-2015 school year.

- 2. Please answer, in one to two sentences, the following questions about the problem situation.
 - a)What problem is the White Oak Community College faculty and students trying to solve?
 - b) EducText, would allow students and faculty to text about assignments, course work, and anything school related for free. Why is this important to the faculty and/or students?
 - c) Why is it important for the Committee to know whether the students send or receive at least 20 text messages each day?

Answers:

(a) Faculty and students are trying to figure out if it would be worth installing a mass text messaging stystem as a form of communication at their school.

(b) Many students don't have latoptops or computers at home, so they can't always check the course websites or email. Students and instructors complain that communication about assignments and course work is challenging.

(c) This would show if people are using text messaging. If they aren't using text messaging, it wouldn't be an effective form of communication for faculty and students at White Oak Community College.

Now, let's collect data about how often students in this class text message.

Note: This activity will involve collecting actual data on texting from students in the class. Hand out small sheets of paper to your students. Ask students to check their cellphones and count the number of text messages that they sent and received the previous day. If they don't have their cell phones with them, ask them to estimate. If students did not send or receive text messages, ask them to put '0'. Collect the information anonymously, and write the data set on the board.

[Student Handout]

3 a) What is the typical number of texts that students in this class send and receive each day? Discuss with your group the best way to approach this question. Try to find a number. Think about how you got that number.

Note: Students will likely calculate the mean, though some may choose to calculate the median. Some may adopt both methods. Here, the goal should be to allow students to decide which would be the more meaningful (or easily calculated) measure. Ask students to explain their method and why those chose the method they chose.

You will probably want to show students results on the board. One option would be to use a boxplot, as it would allow students to see the mean and median on the chart.

[Student Handout]

b) Based on the results of your class, would it be worth buying EducText at WOCC? Should the college purchase EducText?

Note: Discuss the results on the board. If no group calculated the median, do this calculation as a class or ask students to derive this result on their own. Then explore the difference in these two values.

Facilitation Prompts

• How did you find the typical number of texts that students in this class send and receive each day?

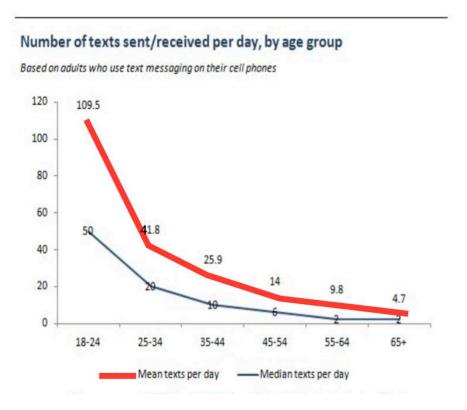
- What methods can we use to determine the typical number of texts that students in this class send and receive each day?
- Why is the mean different from the median?
- If we went to a different class and collected the same kind of data, do you think that the mean would also be bigger (or smaller) than the median?
- Is the mean always bigger (smaller) than the median?

Here, again, the goal is to facilitate student thinking on the difference between mean and median, but not necessarily to come to any conclusions, as the distinction will be more explicitly explored throughout the rest of the lesson.

[Student Handout]

Figure 1 below shows national data about the number of texts adults send and receive. These results are organized by age group.

Figure 1: Number of texts/sent received per day, by age group in the U.S. during 2011



*Based on national data. Source: http://www.pewinternet.org/2011/09/19/how-americans-use-text-messaging/

(4) Examine *Figure 1*. Make sure to take note of the labels in the figure.

a) Write at least two factually precise statements about the data. That is, write two statements that are based on information that is in the data chart above. Write your answer in 1-2 complete sentences. (It is important to write complete sentences because it helps your instructor better understand your mathematical thinking.) Be sure your answer is precise and complete.

Writing Tip!

Writing precise and complete answers in complete sentences can be challenging. Here is an example of a possible answer to this question:

Twenty-five to thirty-four year-old students at WOCC send or receive an average of 41.8 texts per day and a median of 29 texts per day.

Answers will vary. Many students will be inclined to write a general statement about the data, for example, "Older people text less often." However, the purpose of this question is to develop in students the facility for expressing quantitative information precisely and completely. For students who wish to convey that texting frequency decreases with age, emphasize that their statement must state, specifically, by how much texting frequency has declined, over what period. For students who wish to convey that the mean is higher than the median, emphasize that their statement must present the exact quantitative disparity and be specific about age group.

[Student Handout]

b) Now, using *Figure 1* to draw one conclusion about texting and age. Write your answer in 1-2 **complete sentences.**

Note: If students struggle with how to do this, you may want to ask the following leading questions, "How many texts do eighteen to twenty-four year olds send a day?" How many texts do sixty-five year old + send a day?". What kind of conclusions can you draw about this?

Answer: Here students are encouraged to draw general conclusions, most likely concerning the decline of texting by age or the disparity between mean and median. Here precise scientific language is emphasized over quantitative precision. For example, instead of "Older people text less often," students should be encouraged to write, "Texting frequency declines as age increases."

(c) If the texting data shown in *Figure 1* for 18-24 year olds is representative of the texting habits of WOCC's students, would the EducText system be worth the expense?

Note: Make sure students understand the notion of "representative" in this question.

Answer: The median of the data is 50, which means that half of the students at the school send or receive 50 or more texts per day. This means that the majority of students must send or receive more than 20 texts, so the system would be worth the expense.

[Student Handout]

(5) Why do you think the mean number of texts sent and received is significantly greater than the median? Again, write your answer in 1-2 complete sentences.

Answer: This question can be discussed in groups or directly addressed as class discussion. Some students may realize that the disparity owes to outliers on the upper end of the texting frequency spectrum but may have difficulty articulating this. Try to involve as many contributors as possible, facilitating the class's development of this idea without articulating your own explanation. The next question will afford an opportunity for the class to more directly engage this quantitative concept.

[Student Handout]

(6) For the 45-54 year olds, their mean is 14 and their median is 6. Construct a sample of six individuals to illustrate what data set might look like which has a mean of 14 and median of 6. As you are working, think about the process you use to determine this set of numbers.

Note: In lesson 2.6 students were asked to derive a data scenario according to a particular mean or median; this problem is more challenging because it requires students to do both at the same time. If students are struggling, suggest that they "fix" the median first, e.g, "Which data points are involved in fixing the median?" Once they have established the middle two data points (which must average to 50), then advise students that they can now pick the other data points so that the mean is 109.5. In Lesson 2.6, students learned that fixing the mean amounts to making sure that the sum of data values is n times the average, or in this case, 6*109.5 = 657. Here, though, two data values must be less than the smallest middle value, and two must be greater than the largest middle value. One example of such a data set would be 40, 46, 48, 52, 80, 391.

Have several groups present their answers on the board. All answers will necessarily involve one or two data values significantly greater than the others. Develop this idea in class discussion. Here, the term "outlier" might be introduced. Be sure to discuss the different processes students used to determine this sample.

[Student Handout]

(7) What will generally be true for a data set with a significantly greater mean than median?

Answers will vary. One example answer: "If the mean of a data set is significantly greater than the median, there are more outliers that are larger than the median (or mean) value."

(8) Write an example set of five data points for which the mean is significantly *less* than the median.

Answers will vary. One example: 0, 80, 81, 82, 83.

(9) In your research, you decided to learn more about different uses of information technology. You wanted to understand the possible options that might be available for staff and students to communicate at White Oak Community College. You found the following statistics:

- A. Approximately half of all Americans between 18 and 24 send and receive at least 50 texts a day
- B. On average, American 18-29-year-olds send fifty more texts per day than 30-49 year olds.
- C. The best hour of the day to post on Twitter (measured by re-tweets) is at 5pm.
- D. The average amount of time spent per Facebook visit is 20 minutes.
- E. 49% of 18-24-year-olds go to Facebook when they wake up.
- F. Among all social media sites Facebook has the most users.
- G. The average Facebook user is 40.5 years old.
- H. Half of all Facebook users age 18-29 have at least 300 Facebook friends.
- For each of the above statistics, decide whether the mean, median, mode is being used. If neither the mean, median, or mode is being used, write "neither."
 - 9 (A):
 - 9 (B):
 - 9 (C):
 - 9 (D):
 - 9 (E):
 - 9 (F):
 - 9 (G):
 - 9 (H):

Answers:

A. median B. mean C. mode D. mean E. neither F. mode G. mean H. median.

Note: Students may find it difficult to distinguish mean from median. Here, the key point is that the median divides any data set into two halves. In the case of C and F, the mode is involved because there is category that is "coming up" more often than any other (hour of the day, Facebook). Part E is tricky, but it is not the mean, since an average is not being calculated, it's not a median, because there are not data values that can be divided into "upper" and "lower" regions, and it's not clearly a mode, since it's not clear that this category (what people do when they wake up) "happens" more than any other.

Making Connections

Record the important mathematical ideas from the discussion.

Further Applications

Broadband Speeds

Net neutrality refers to the idea that information should be processed through the Internet in the order received, rather than allowing companies to pay more to let their traffic move through the Internet more quickly. This is an issue because the Internet is limited in how much information it can deliver, and how fast. Some people are frustrated by slow download speeds for media, such as Netflix streaming.

In studying this issue, you found some surprising information. You found following statistic:

The median internet usage in the U.S. is less than 2 gigabytes per month per person, while the mean usage was over 9 gigabytes per month per person.

(1) Does this suggest that people are consuming approximately equal amounts of the Internet, or are some people consuming much greater amounts than others? Explain your answer in one or two complete sentences.

Answer:

Students can be expected to come up with many different kinds of explanations. Solicit them in class discussion or have several students come up and write their answers on the board. A sample (good) answer might be like the following: "The mean is much bigger compared to the median, which means that while most users use around 2 gigabytes per month, there are a few users who use much more than the rest, which brings up the average."

Some students may have difficulty figuring this out. The following facilitation prompts may be helpful:

What does the fact that the median is 2 tell us? (Half of the people in the country use 2 or fewer gigabytes). How about the other half? Since the lower half are from zero to 2, maybe the upper half is from 2 to 4? (No, because the average is 9). 9 is the average of 2 and what number? (16). So maybe the other half are using about 16 gigabytes? If the lower half were using about 2, and the upper half were using about 16, wouldn't the average be 9? (yes, but then the median would also be 9, etc.)

Some students may continue to struggle after the discussion. The following exercise will help these students to make sense of the analysis:

[Student Handout]

(2) Construct a scenario of five download speeds in which the median of the five speeds is 2 gigabytes and the mean is 9 gigabytes.

Answer: There are an infinite number of possible answers. One example: 0,1,2,2,40. Have several groups come to the board to present their solutions.

Website Analytics

Your company maintains a website to sell its products. You wish to analyze the activity your website has received. You measure the activity in terms of "visits" to your site. You want to report to the president of the company about the activity of the website. You find that on most days, your site received between 8,000 and 12,000 visits. However, there were 8 days during the year in which the website was offline due to construction. Also, during the first month, the website had not yet been advertised on Google searches, so the number of visits was very small.

(3) Which will be greater, the mean or the median number of visits per day? What would offer the better representation of how active the website has been over the last year - the mean or the median number of visits per day? Explain your answer in one or two complete sentences.

Answer: The data set consists of values that are mostly in the range of 8,000 to 12,000; however, there are a number of values that are well below this range due to factors that are not typical, nor representative of the website's activity. These are "outliers" and while they will reduce the mean significantly, they will not significantly affect the median. Hence the median better represents the general accessibility of the website.

(4) Construct a scenario of 5 values in which the median is 10,000 visits but the mean is 8,000 visits.

Answer: There are an infinite number of possible answers. One example: 0,8000,10000,11000,11000. Encourage students to think of this problem as a kind of puzzle. They know what the middle value must be, and they know that since the mean is 8000, the values must add up to 40000 (drawing on lesson 2.6).

Key to the OCE

(1) iii

(2) (a) The median house price in 2007 was about \$248,000. This means that about half the houses cost \$248,000 or less and about half cost \$248,000 or more.;

(b) Answers will vary. Possible answers are that both show median house prices for the U.S. 2.1.4 data are for new houses 1999 to 2010; 2.7 are for new homes 1963–2008. 2.7 also shows average or mean prices. Both are line graphs and show trends in the data.

- (3) \$2,450
- (4) \$3,200
- (5) (a) iii; (b) i; (c) ii
- (6) (a) 4; (b) i; (c) 40; (d) 11; (e) 9; (f) 8; (g) ii
- (7) (a) B; (b) A; (c) A; (d) C
- (8) (a) i; (b) ii; (c) \$77,600; (d) Yes; (e) France and the United States; (f) i and ii
- (9) (a) iii; (b) ii; (c) iv
- (10) (a) The Consumer Price Indexes (CPI) program produces monthly data on changes in the prices paid by urban consumers for a representative basket of goods and services. (b) 87%; (c) Yes;
 (d) No; (e) ii; (f) i; (g) "As an economic indicator" or "as a deflator of other economic series" or "as a means of adjusting income payments"
- (11) (a) \$2.10; (b) 40.8%
- (12) i
- (13) ii
- (14) Answers will vary.
- (15) Answers will vary.
- (16) Answers will vary.

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dent success. The original version of this work, version 1.0, was created by The Charles A. Dana Center at The University of Texas at Austin under sponsorship of the Carnegie Foundation for the Advancement of Teaching. This version and all subsequent versions result from the continuous improvement efforts of the Carnegie Networked Improvement Community. The network brings together community college faculty and staff, designers, researchers and developers. It is a research and development community that seeks to harvest the wisdom of its diverse participants through systematic and disciplined inquiry to improve developmental mathematics instruction. For more information on the Quantway[®] Networked Improvement Community, please it carnegiefoundation.org.

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