COURSE COMPETENCIES

1. **Factor algebraic expressions**
   - You factor using the greatest common factor
   - You factor binomials and trinomials
2. **Solve quadratic equations over the set of real numbers**
   - You identify coefficients of a quadratic equation in standard form
   - You solve second-degree equations to determine all relevant solution(s).

EXPLICIT CONNECTIONS

It is important that each person understands how to solve second-degree equations. Higher degree equations are frequently used in circuit analysis.

NOTES TO SELF

- Encourage each student to check his or her answers. They just do not want to take the time to check their answers.
- Encourage each student to use the quadratic formula.

Duration Minutes	Lesson	Suggested Structure
10	Lecture - Polynomial Introduction	Cohort
15	Problem Set 6.1 - Parabolas and Polynomials	Group
15	Problem Situation 6.2 – Polynomial Addition and Subtraction	Group
20	Blackboard - Practice Set 1 - Polynomial Add/Sub	Individual
20	Lecture - Polynomial Multiplication	Cohort
15	Problem Situation 6.3 – Polynomial Multiplication	Group
20	Problem Situation 6.4 – Magic Rectangle	Group
10	Blackboard - Practice Set 2 - Polynomial Multiplication	Individual
25	Lecture - Second degree equations	Cohort
20	Problem Situation 6.5 – Quadratic Equations and the Quadratic Formula	Group
15	Blackboard - Practice Set 3 - Quadratic Eqn	Individual
20	Lecture - Factoring	Cohort
20	Problem Situation 6.6 – Polynomial Factoring	Group
15	Blackboard - Practice Set 4 - Polynomial Factoring	Individual
15	Problem Situation 6.7 – Polynomial Division	Group
10	Blackboard - Practice Set 5 - Polynomial Division	Individual
20	Quiz	Cohort

Lesson	***Objectives***	***Material***
6.1	Polynomial Introduction	Polynomials and Parabolas
6.2	Polynomial Addition and Subtraction	Polynomial Addition and Subtraction
6.3	Polynomial Multiplication	Polynomial Multiplication
6.4	Application of a Polynomials	Magic Rectangle
6.4	Quadratic Equations	Quadratic Equations and Formula
6.5	Polynomial Factoring	Polynomial Factoring
6.6	Polynomial Division	Polynomial Division

Prerequisite Assumptions

Before beginning the lesson, students should understand and be able to;

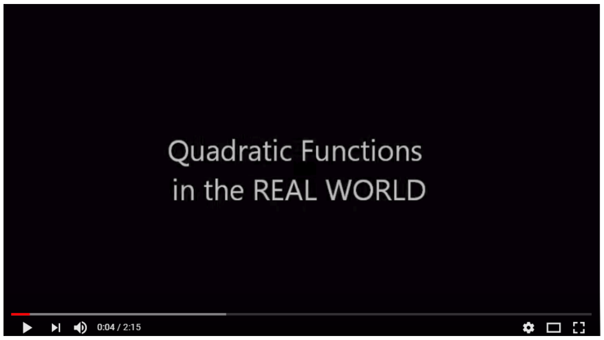
✓ Solve linear equations
✓ Solve systems of equations with 2 unknowns

Specific Objectives

By the end of this lesson, you should be able to;

✓ Define polynomials
✓ Simplify polynomials
✓ Add and subtract polynomials
✓ Multiply polynomials
✓ Factor polynomials
✓ Solve second degree equations using the quadratic equation

Problem Situation 6.1 – Polynomials and Parabolas



https://youtu.be/He42k1xRpbQ?t=4

Polynomial Overview

Poly means *many* and **nomial** means *names* or *terms.* Thus, a **polynomial** is a mathematical expression that has many *terms*. A **term** is either a number or variable, or numbers and variables multiplied together. When terms are separated by a **+** (plus) or **–** (minus) sign, all the terms together form a polynomial.

A polynomial itself is named or *characterized* by the terms, specifically

- The value of the exponents which must be integers and greater than 0,
- The number of terms in the polynomial.

A polynomial can have from one term to many, many terms. We often refer to polynomials with:

- one term as a *monomial,*
- two terms as a *binomial,*
- three terms as a *trinomial.*

A polynomial is also identified by the term with the highest *exponent* value:

- An exponent of one is a *first-degree* polynomial.
- An exponent of two is a *second-degree* polynomial.
- An exponent of three is a *third-degree* polynomial.
- An exponent of four is a *fourth-degree* polynomial, etc.

A polynomial with a term such as x^2y^3, with more than one variable, the exponents are "added" together to get the degree of the term. Therefore, this term is a fifth-degree polynomial, xy is a second-degree polynomial and xy^2 is a third-degree polynomial.

Polynomial expressions should be written in **standard form**, which means the terms of the polynomial are written with the highest value exponents on the left and the lowest value exponents on the right. Some examples of polynomials in standard form are:

$$4x^2 + 2 \qquad\qquad y^4 - 5xy + y$$

1) For each expression answer the following
   - ✓ *Polynomial?*

   If it is a polynomial answer the following
   - ✓ *Number of terms:*
   - ✓ *Degree:*
   - ✓ *Identify the coefficient of the highest degree term:*

   a) $x^4 + y^3 + xy^2 - x - 3$
   - ✓ *Polynomial?* Yes
   - ✓ *Number of terms:* 5
   - ✓ *Degree:* 4
   - ✓ *Identify the coefficient of the highest degree term:* 4

   b) $2x^2$
   - ✓ *Polynomial?* Yes
   - ✓ *Number of terms:* 1
   - ✓ *Degree:* 2
   - ✓ *Identify the coefficient of the highest degree term:* 2

   c) $-y^3 + x^{-2} + 12$
   - ✓ *Polynomial?* No
   - ✓ *Number of terms:*
   - ✓ *Degree:*
   - ✓ *Identify the coefficient of the highest degree term:*

   d) $x^3 - x^2 + x$
   - ✓ *Polynomial?* Yes
   - ✓ *Number of terms:* 3
   - ✓ *Degree:* 3
   - ✓ *Identify the coefficient of the highest degree term:* 1

Problem Situation 6.2 – Polynomial Addition and Subtraction

To **simplify** a polynomial, one must combine *like* terms. A **like** term contains the *same variables* with the *same exponents*. *($4x^2 + 3x^2 = 7x^2$ and $6xy^3 - 2xy^3 = 4xy^3$)*

1) For the following polynomials combine like terms and put into standard form.

   a) $2x^3 + 4y^2 - 3xy + 3y^2 - 4 - x^3 + 7x^2 + 5$

$(2x^3 - x^3) + (4y^2 + 3y^2) + 7x^2 - 3xy + (5 - 4)$

$x^3 + 7y^2 + 7x^2 - 3xy + 1$

b) $4x^2 + 2y^2 + 6xy - 10 - 6x^2 + 3x^2 + 7xy + 4$

$(4x^2 + 3x^2 - 6x^2) + 2y^2 + (6xy + 7xy) + (-10 + 4)$
$x^2 + 2y^2 + 13xy - 6$

c) $4 - 6r^2 + 2n + 13rn - n - 9r^2 + 2n^2 + rn + 12$

$(-6r^2 - 9r^2) + 2n^2 + (13rn + rn) + (2n - n) + (12 + 4)$
$-15r^2 + 2n^2 + 14rn + n + 16$

2) Perform the computation and simplify the following polynomials.

a) $(a^2 + 4a + 2) + (3a^2 - 6)$

$(a^2 + 3a^2) + 4a + (2 - 6)$
$4a^2 + 4a - 4$

b) $(b^3 - 3b^2 + 6) - (-5b^2 + b - 4)$

$b^3 + (-3b^2 + 5b^2) - b + (4 + 6)$
$b^3 + +2b^2 - b + 10$

c) $(2x^2 - 6) - (4x^2 - x)$

$(2x^2 - 4x^2) + x - 6$
$-2x^2 + x - 6$

<u>Problem Situation 6.3 – Polynomial Multiplication</u>

Use the *distributive* property to multiply a *monomial* by a *polynomial*

<u>Examples:</u>

$2(4x^2 + 2)$ $\quad$ *distribute 2 by multiplying each term of the polynomial by 2.*

$(2 * 4)x^2 + (2 * 2) = 8x^2 + 4$

$2x(y - x + 2)$ $\quad$ *distribute 2x by multiplying each term of the polynomial by 2x.*

$(2x * y) - (2x * x) + (2x * 2) = 2xy - 2x^2 + 4x \rightarrow standard\ form\ \ -2x^2 + 2xy + 4x$

1) Perform the following operations, simplify the polynomials, and put them in standard form.

a) $x(x - 2)$

I will distribute the x. $\quad (x * x) - (2 * \mathrm{x}) = x^2 - 2\mathrm{x}$

b) $\mathrm{y}^2(\mathrm{y} - \mathrm{x} + 3)$

I will distribute the y^2. $\quad (y^2 * y) - (y^2 * \mathrm{x}) + (y^2 * 3) = y^3 - y^2\mathrm{x} + 3y^2$

c) $mn(n + m)$

I will distribute the mn. $\quad (mn * n) + (\mathrm{mn} * \mathrm{m}) = mn^2 - nm^2$

Multiplying a polynomial by a *binomial* is done using the distributive property.

Examples:

$(7x-2)(x+3) \rightarrow \quad 7x(x+3)-2(x+3)$
$7x^2+21x-2x-6$
$7x^2+19x-6$

$2(4y\ +\ 3)(5y - 3) \rightarrow \quad 2[4y(5y-3)+3(5y-3)]$
$2[20y^2-12y+15y-9]$
$2[20y^2+3y-9]$
$40y^2+6y-18$

2) Perform the following operations, simplify and put in standard form.

a) $(-\mathrm{x}+2)(2\mathrm{x}+6)$

Use the distributive property.
$-\mathrm{x}(2\mathrm{x}+6)+2(2\mathrm{x}+6)$
$-2\mathrm{x}^2-6\mathrm{x}+4\mathrm{x}+12$
$-2\mathrm{x}^2-2\mathrm{x}+12$

b) $3(y-3)(-3y+2)$

$3[y(-3y+2)-3(-3y+2)]$
$3[-3y^2+2y+9y-6]$
$3[-3y^2+11y-6]$
$-9y^2+33y-18$

c) $(x+4)(x-4)$

$x(x-4)+4(x-4)$
$x^2-4x+4x-16$
x^2-16

d) $2(4y-6)(-y+3)$

$2[4y(-y+3)-6(-y+3)]$
$2[-4y^2+12y+6y-18]$
$2[-4y^2+18y-18]$
$-8y^2+36y-36$

e) $(G-3)^2$

$(G-3)(G-3)$
$G(G-3)-3(G-3)$
$G^2-3G-3G+9$
G^2-6G+9

f) $(f\ +\ 2)(f^{\,2}+3f\ +\ 5)$

$f\,(f^2\ +3f\ +\ 5)+2(f^2\ +3f\ +\ 5)$
$f^3\ +3f^2\ +\ 5f+2f^2\ +6f\ +\ 10$
$f^3\ +5f^2\ +\ 11f\ +\ 10$

Problem Situation 6.4 – Magic Rectangle

1) Act 1: Magic Rectangle – What do you notice? What do you wonder?

2) Predict when do you think the area is the largest?

Show the students this Act 1.1 video for a visual.

3) What do you need to know to determine the largest area?

This Act 2 video shows some of the sizes of the rectangle.

4) What are the dimensions of the rectangle that contains the largest area?

$Area = Width * Length \rightarrow Area = (85 + 5x)(62 - 2x) = -10x^2 + 140x + 5270$

5) Act 3 video – The answer revealed.

Problem Situation 6.5 – Quadratic Equations and the Quadratic Formula

A **quadratic** equation is a *second-degree* equation containing a term with an exponent of 2 and no higher degree term with a standard form of:

$ax^2 + bx + c = 0$

There are a number of methods used to determine the *solution* for a second-degree equation. The **Quadratic Formula** is one method.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example: $x^2 - 3x - 4 = 0$

Identify the coefficients: $\boldsymbol{a = 1, b = -3, c = -4}$ Properly insert into the formula and solve for *x*:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x = \frac{3 \pm \sqrt{(-3)^2 - (4 * -4)}}{2} = \frac{3 \pm \sqrt{(9 + 16)}}{2}$$

$$\frac{3 \pm 5}{2} = \frac{8}{2}\ and\ \frac{-2}{2} = 4\ and - 1$$

$Check$: $4^2 - 3(4) - 4 = 0$ √ and $-1^2 - 3(-1) - 4 = 0$ √

Notice that x has two (2) solutions. *Does this make sense?*

1) Use the quadratic formula to determine the solutions for the following second-degree quadratic equations.
   a) $2x^2 + 5x - 3 = 0$

To use the quadratic formula, I need to identify the values of the coefficients.
$a = 2, b = 5, c = -3$

$$x = \frac{-5 \pm \sqrt{5^2 - (4 * 2 * -3)}}{2 * 2} = \frac{-5 \pm \sqrt{25 + 24}}{4} = \frac{-5 \pm 7}{4} = \frac{-12}{4}, \frac{2}{4} = -3, 0.5$$

Check my answers.
- ✓ $2(-3)^2 + 5(-3) - 3 = 0 \; and \; 2(0.5)^2 + 5(0.5) - 3 = 0$

   b) $-x^2 + 3x - 2 = 0$

To use the quadratic formula, I need to identify the values of the coefficients.
$a = -1, b = 3, c = -2$

$$x = \frac{-3 \pm \sqrt{3^2 - (4 * -1 * -2)}}{2 * -1} = \frac{-3 \pm \sqrt{9 - 8}}{-2} = \frac{-3 \pm 1}{-2} = \frac{-4}{-2}, \frac{-2}{-2} = 2, 1$$

Check my answers.
- ✓ $-(1)^2 + 3(1) - 2 = 0 \; and \; -(2)^2 + 3(2) - 2 = 0$

   c) $x^2 + 3x - 3 = 0$

To use the quadratic formula I need to identify the values of the coefficients.
$a = 1, b = 3, c = -3$

$$x = \frac{-3 \pm \sqrt{3^2 - (4 * 1 * -3)}}{2 * 1} = \frac{-3 \pm \sqrt{9 + 12}}{2} = \frac{-3 \pm \sqrt{21}}{2} = -3.79, 0.7913$$

Check my answers.
- ✓ $(-3.79)^2 + 3(-3.79) - 3 = 0 \; and \; (0.7913)^2 + 3(0.7913) - 3 = 0$

   d) $4x^2 + 8x - 12 = 0$

To use the quadratic formula, I need to identify the values of the coefficients.
$a = 4, b = 8, c = -12$

$$x = \frac{-8 \pm \sqrt{8^2 - (4 * 4 * -12)}}{2 * 4} = \frac{-8 \pm \sqrt{256}}{8} = \frac{-8 \pm 16}{8} = \frac{-24}{8}, \frac{8}{8} = -3, 1$$

Check my answers.
- ✓ $4(-3)^2 + 8(-3) - 12 = 0 \; and \; 4(1)^2 + 8(1) - 12 = 0$

   e) $-x^2 - 3x + 10 = 0$

To use the quadratic formula, I need to identify the values of the coefficients.
$a = -1, b = -3, c = 10$

$$x = \frac{3 \pm \sqrt{3^2 - (4 * -1 * 10)}}{2 * -1} = \frac{3 \pm \sqrt{49}}{-2} = \frac{3 \pm 7}{-2} = \frac{-4}{-2}, \frac{10}{-2} = 2, -5$$

Check my answers.
- ✓ $-(2)^2 - 3(2) + 10 = 0 \; and \; -(-5)^2 - 3(-5) + 10 = 0$

f) $2x^2 + 3x - 2$

To use the quadratic formula, I need to identify the values of the coefficients.
$a = 2, b = 3, c = -2$

$$x = \frac{-3 \pm \sqrt{8^2 - (4 * 2 * -2)}}{2 * 2} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4} = \frac{-8}{4}, \frac{2}{4} = -2, 0.5$$

Check my answers.

- ✓ $2(-2)^2 + 3(-2) - 2 = 0 \; and \; 2(0.5)^2 + 3(0.5) - 2 = 0$

Problem Situation 6.6 – Polynomial Factoring

Factoring polynomials is the *opposite* of multiplying terms together, that is, we are *breaking up* the polynomial into terms.

In creating the polynomial we used the multiplication and the distributive property to get the individual terms. For example, $3(x + 1) = 3x + 3$. When factoring, we want to look for and *extract* the common elements, really using the distributive property 'backwards'. For example, the 3 is common so we'll pull it out: $3x + 3 = 3(x + 1)$.

Notice that by using multiplication we can 'see' how the polynomial terms are related to its factors:

$(x + a)(x + b) = x^2 + bx + ax + ab = x^2 + x(a + b) + ab$

Examples:

$4x^2 + 8x - 32 = 0$

Start by 'pulling' 4 out of the polynomial: $4(x^2 + 2x - 8) = 0$

Using the property above we notice that $a + b = 2 \; and \; ab = -8$

This means that either a or b must be negative for the product 'ab' to be negative.

We can solve for a and b such that if $\boldsymbol{a} = \mathbf{4}$ and $\boldsymbol{b} = \mathbf{-2}$, $4 + (-2) = 2$ and $4 * -2 = -8$

So we get as our factored polynomial: $4(x + 4)(x - 2) = 0$

This will **ONLY** be true if, x + 4 = 0, that is **x = -4** or if x – 2 = 0, that is, **x = 2**.

Check that the solutions for x will make the original equation true.

$4(-4)^2 + 8(-4) - 32 = 0$ √ and $4(2)^2 + 8(2) - 32 = 0$ √

$x^2 - 2x - 24 = 0$

$(x + a)(x + b) = 0$

$a + b = -2 \; and \; a * b = -24$

$a = 4 \; and \; b = -6$

$(x + 4)(x - 6) = 0$

$Solution: \boldsymbol{x} = \mathbf{-4} \; and \; \boldsymbol{x} = \mathbf{6}$

$(-4)^2 - (2 * -4) - 24 = 16 + 8 - 24 = 0$ √

$6^2 - 2 * 6 - 24 = 36 - 12 - 24 = 0$ √

$x^2 - 9x + 20 = 0$

$(x + a)(x + b) = 0$

$a + b = -9 \; and \; a * b = 20$

$a = -5 \; and \; b = -4$

$(x - 5)(x - 4) = 0$

$Solution: \boldsymbol{x} = \mathbf{4} \; and \; \boldsymbol{x} = \mathbf{5}$

$4^2 - (9 * 4) + 20 = 16 - 36 + 20 = 0$ √

$5^2 - (9 * 5) + 20 = 25 - 45 + 20 = 0$ √

1) Factor each of the following equations.

a) $x^2 - x - 12 = 0$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = -1$ *and* $a * b = 12$ *When* $a = -4$ *and* $b = 4$ *this is true.*
$(x-4)(x+3) = 0$
Solution: $\boldsymbol{x = 4}$ **and** $\boldsymbol{x = -3}$ $4^2 - 4 - 12 = 0$ *and* $(-3)^2 - (-3) - 12 = 0$

b) $y^2 + 10y = -25$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = 10$ *and* $a * b = 25$ *When* $a = 5$ *and* $b = 5$ *this is true.*
$(x+5)(x+5) = 0$
Solution: $\boldsymbol{x = -5}$ $(-5)^2 + 10(-5) + 25 = 0$

c) $x^2 + 13x + 42 = 0$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = 13$ *and* $a * b = 42$ *When* $a = 6$ *and* $b = 7$ *this is true.*
$(x+6)(x+7) = 0$
Solution: $\boldsymbol{x = -6}$ **and** $\boldsymbol{x = -7}$ $(-6)^2 + 13(-6) + 42 = 0$ *and* $(-7)^2 + 13(-7) + 42 = 0$

d) $x^2 - 9 = 0$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = 0$ *and* $a * b = -9$ *When* $a = 3$ *and* $b = -3$ *this is true.*
$(x+3)(x-3) = 0$
Solution: $\boldsymbol{x = -3}$ **and** $\boldsymbol{x = 3}$ $(3)^2 - 9 = 0$ *and* $(-3)^2 - 9 = 0$

e) $3x^2 + 3x = 18$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = 1$ *and* $a * b = -6$ *When* $a = -2$ *and* $b = 3$ *this is true.*
$3(x-2)(x+3) = 0$
Solution: $\boldsymbol{x = 2}$ **and** $\boldsymbol{x = -3}$ $3(2)^2 + 3(2) = 18$ *and* $3(-3)^2 + 3(-3) = 18$

f) $2x^2 - 4x - 30 = 0$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = -2$ *and* $a * b = -15$ *When* $a = -5$ *and* $b = 3$ *this is true.*
$2(x-5)(x+3) = 0$
Solution: $\boldsymbol{x = 5}$ **and** $\boldsymbol{x = -3}$ $2(5)^2 - 4(5) - 30 = 0$ *and* $2(-3)^2 - 4(-3) - 30 = 0$

g) $x^2 + 8x + 16 = 0$

Since this is a second-degree quadratic equation, I know that $(x+a)(x+b) = 0$.

$a + b = 8$ *and* $a * b = 16$ *When* $a = 4$ *and* $b = 4$ *this is true.*
$(x+4)(x+4) = 0$
Solution: $\boldsymbol{x = -4}$ $(-4)^2 + 8(-4) + 16 = 0$

Problem Situation 6.7 – Polynomial Division

Dividing polynomials can be accomplished in two ways: by straight division or by factoring.

Factoring

$$\frac{x^2-3x+2}{x-1}$$

$$\frac{x^2-3x+2}{x-1}=\frac{\cancel{(x-1)}(x-2)}{\cancel{x-1}}=\boldsymbol{x-2}$$

1) Divide the following polynomials

a) $\frac{x^2-5x-36}{x+4}$

$$Factoring: \frac{x^2-5x-36}{x+4}=\frac{\cancel{(x+4)}(x-9)}{\cancel{x+4}}=\boldsymbol{x-9}$$

b) $\frac{2x^2+4x-6}{x+3}$

$$Factoring: \frac{2(x^2+2x-3)}{x+3}=\frac{2\cancel{(x+3)}(x-1)}{\cancel{x+3}}=2(\boldsymbol{x-1})$$

c) $\frac{2x^2+2x-12}{2x+6}$

$$Factoring: \frac{2(x^2+x-6)}{2(x+3)}=\frac{2\cancel{(x+3)}(x-2)}{2\cancel{(x+3)}}=\boldsymbol{x-2}$$