Lesson: Distributive Property Mathematics for Elementary Teachers I

Instructor Notes Thomas Geil

**Goals**:

* Distributive Property of Multiplication over Addition

**Prerequisite Knowledge**:

* Meaning of multiplication as taught in this course
* Basic multiplication facts for zero through ten
* Understand multiplication syntax: (use of ×, \*, ·, or parenthesis)

**Lesson Materials:**

* One inch tiles, 4 colors (Note – grid paper can be substituted for tiles for the entire lesson)
* Four different colored pencils, crayons, markers, or pens
* Grid paper
* Printed handout (Student notes) for each student (I have included two versions of student notes)
* Tape

**Lesson Breakdown:**

|  |  |  |
| --- | --- | --- |
| **Activity** | **Size of Group** | **Time in Activity****(Total Time – 75-105 minutes)** |
| Form groups of 3-4 and distribute one bag of each color per group | Groups of 3-4 | 5 minutes |
| Quick review of last class activityEx 4(3) Ex 5(2) using tiles | Solve individually within the group and then compare to others within group. Discuss as whole class | 5 minutes |
| “Use tiles to model the problem 5(17) and create an expression represented by the area model” | Have them discover it will take pairs within the group, group work, then whole class | 10-15 minutes |
| “Use the tiles to model 9(15) and to create an equation represented by the area model” | Pairs within group, discussion within group, then whole class | 10-15 minutes |
| “Use the tiles to model 14(13) and write an equation represented by the area model” | Have them discover it will take the group of 4, then whole class discussion | 15-20 minutes |
| “Use the grid to create an area model for (27)(35) and an equation represented by the area model” | Group of 4, then whole class | 10-15 minutes |
| “Discuss how can the area model for (127)(35) be created by expanding on the #6 example” | Group of 4, then whole class | 10 minutes (Optional) |
| “Discuss within your group how an area model could be used to simplify (x + 4)(x + 3) and draw a rough idea of what that model would look like” | Individual, within group, then whole class | 10 minutes |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

Lesson: Distributive Property Mathematics for Elementary Teachers I

Instructor notes:

**Goals**:

* Distributive Property of Multiplication over Addition

**Prerequisite Knowledge**:

* Meaning of multiplication as taught in this course
* Basic multiplication facts for zero through ten
* Understand multiplication syntax: (use of ×, \*, ·, or parenthesis)

NOTE: This lesson should involve very little whole class instruction. Let the students discover and experiment within the groups. Pose each question and let them discuss within their group. Walk around the room and listen to the usage of the language of math. Ask questions individually when students are off track. Try to let them discover when they need more tiles, or to combine colors.

1. Form groups of 3-4 students depending on class size. Hand out bags of tiles to each group (or stacks of grid paper). Each group should receive 50 tiles each of 4 different colors. If you do not have access to tiles you can do the entire lesson using the grid paper included with this lesson. Students come late to class. It is possible to have them join in or separate already formed groups. This is an issue with group work that takes the whole class period.
2. Quick review from last class. Have each of the students select a bag of tiles.

Pose the problem “Given your bag of tiles, show an area model (array diagram) representing 4(3) and 5(2)”. Have the students discuss the models created within their group. Have two (or more) students post drawings. Stress that we are using the book definition where groups are shown vertically and objects are shown horizontally. If a student creates 3(4) or 2(5) in area model post that too then discuss the Commutative Property and that other books may have a different way to show multiplication. Stress this is an area model so the tiles should be touching each other. This will help drawing things later in the lesson.

1. Pose the problem “Use tiles to model the problem 5(17) and create an expression represented by the area model”. Tell the students that each rectangle used must use only one color of tiles and once you use a color you can’t use it again.

They will not be able to do this with one color as it is more than 50. Let the groups come to this conclusion. Walk around the room and guide those struggling to start, or groups using irregular models. Allow mistakes to lead to discussion within the groups. These could include a single rectangle made up of multicolored sections, rectangles that are not solid, L-shaped regions). The goal is to have two of the students join together and use two different colors to create the area model consisting of two single colored rectangular regions. There are multiple answers. Some possible correct ways would be showing the following models:

* 5(8) and 5(9) using 2 colors

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

* 5(7) and 5(10) using 2 colors

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

* 2(17) and 3(17) using 2 colors

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

* Some groups could even join 3 or 4 colors and that would be ok. Ex 5(5) and 5(5) and 5(7) using 3 colors

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Possible error type regions:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Have several groups post their models on the board and discuss the results and the expressions. Try to see if they can create the expressions 5(10 + 7) or 5(10) + 5(7) or whatever expressions that fit with their model. Have the students discuss how the number of groups, 5, and the number of objects, 17, are represented in the expressions (or the values they used). Turn this into an equation 5(17) = 5(10 + 7) = 5(10) + 5(7) = 50 + 35 = 85. Listen for someone to call that the Distributive Property. Lead them to it if they don’t. Have them use colors to relate the area model to the distributive property expression they created.

1. Pose the problem “Use the tiles to model 9(15) and to create an equation represented by the area model”. Drift around the room from group to group allowing mistakes. Point out that we are once again building single colored solid rectangles and once a color is used it cannot be used again. There are many possible models. Some possible are as follows:
* (5 + 4)(10 + 5)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

* (6 + 3)(7 + 8)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Have groups post the results. The possible models and equations will vary - first one above (5 + 4)(10 + 5) = 5(10+5) + 4(10+5) = 5(10)+5(5)+4(10)+4(5) = 50 + 25 + 40 + 20 = 135. Listen for the groups to talk about “FOIL”. Bring up the idea if nobody does. Some may also do this with only 3 colors of tiles and use vertical or horizontal regions.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

1. Pose the problem “Use the tiles to model 14(13) and write an equation represented by the area model”.
* (7 + 7)(6 + 7)

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

Let them work on this for a while. This is the first one that will require all 4 colors in the FOIL pattern. Watch that within the group they discuss how the numbers in the first set of parenthesis represent the 14 groups and the numbers in the second set represent the 13 objects in each of those groups. Listen to the discussion within the groups as they develop equations represented in their area models. Post the equations. Now give them the grid paper and ask them to use numbers that have multiplication facts the kids would be more successful working with. See if any of the groups created the solution (10 + 4)(10 + 3) = . Ask if there is anything about this model that is beneficial. This is the way we will want them to do the rest of the problems.

1. Pose the problem “Use the grid paper to create an area model for (27)(35) and an equation represented by the area model”. Look for them to use the expression (20 + 7)(30 + 5) to once again create easier multiplication problems. Use tape to combine grids. Have groups hold up their models.
2. (OPTIONAL depending on time). Pose the question “How can the area model for (127)(35) be created by expanding on the #6 example”. Have them draw a rough sketch not exactly to scale and post the results on the board.
3. Algebra Extension – Say “Discuss within your group how an area model could be used to determine the result of (x + 4)(x + 3) and to draw a rough idea of what that model would look like”. Listen to see if any group members connect this problem to 14(13) and how if x = 10 we have found the result.

Lesson: Distributive Property Mathematics for Elementary Teachers I

Student notes:

**Goals**:

* Distributive Property of Multiplication over Addition

**Prerequisite Knowledge**:

* Meaning of multiplication as taught in this course
* Basic multiplication facts for zero through ten
* Understand multiplication syntax: (use of ×, \*, ·, or parenthesis)
1. Form groups of 3-4 students depending on class size.
2. Quick review from last class. Show an area model representing 4(3) and 5(2).
3. Create an area model for the problem 5(17).

Write an expression represented by your area model

1. Create an area model for 9(15).

Write an equation represented by your area model

1. Create an area model for 14(13).

Write an equation represented by your area model

1. Create an area model for 27(35).

Write an equation represented by your area model

1. Use #7 to create a rough sketch of an area model for 127(35).

Write an equation represented by your area model

1. What area model could be used to represent (x+4)(x+3)?

Write an equation represented by your area model

Lesson: Distributive Property Mathematics for Elementary Teachers I

Student notes:

**Goals**:

* Distributive Property of Multiplication over Addition

**Prerequisite Knowledge**:

* Meaning of multiplication as taught in this course
* Basic multiplication facts for zero through ten
* Understand multiplication syntax: (use of ×, \*, ·, or parenthesis)

Lesson: Distributive Property Mathematics for Elementary Teachers I

Student notes: