Lesson: Properties of Logarithms College Algebra

Instructor notes: Thomas Geil

Goals:

- Develop the properties of Logarithms

Prerequisite Knowledge:

- Use of a calculator
- Basic arithmetic facts for zero through ten
- Understand multiplication syntax: (use of ×, *, ·, or parenthesis)

Lesson Materials:

- Calculator with log and ln functions
- Writing utensil
- Printed student handout

Lesson Breakdown:

Activity	**Size of Group**	**Time in Activity** **(Total Time 60 –80 minutes)**
Form groups and hand out student note sheet.	2-3 students per group	5 minutes
Give some calculator directions for those not familiar with the location of the LOG and LN buttons.	While in groups	5 minutes
Have students complete pages 2 and 3 of the handout and discuss graphs.	Group work, whole class	20-25 minutes
Hand out page 4 of the student packet and have the students post the observations on the board.	Group work, whole class	10-15 minutes
Hand out page 5 of the student packet and have the students create properties from the observations and post properties on the board.	Group work, whole class	10-15 minutes
Hand out page 6 of the student packet and ask students to use properties and examples to complete the work.	Group work, whole class	10-15 minutes

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*****YOU CAN SWITCH THE ORDER OF THE HANDOUTS TO FIT HOW YOU TEACH THE SUBJECT OF LOGS AND INVERSES.**

1. Form groups of 2-3 students depending on class size. Start with 2 in a group for late comers to join already formed groups.

2. Help students locate the LOG and LN button on the various calculators being used. Talk about what base each of these functions represents.

3. Hand out page 2 and 3 of the student packet. By handing each page out individually students will not be able to move ahead of the class or see what is coming. Watch that students complete graphs with and without using the graphing capabilities of the calculator. Have groups sketch graphs on the board. Discuss Domain and Range of the exponential functions.

4. Hand out page 4. Have the students complete the 28 questions and list as many observations as they can. Post the observations on the board. Examples could be log of negatives, log of 0, log of positives less than 1, log of 1, sum of two logs is equal to the log of the product, difference of two logs is equal to the log of the quotient, powers can come in from of a log, e is 2.71828 …. This may take a while. Post as many of the ideas as possible. Looking for the properties of logs. Have groups post the answers for the 4 problems at the bottom of the page.

5. Hand out page 5 of the student packet. Give time for students to investigate the notation of logs and the connection to exponentials.

6. Hand out page 6 of the student packet. Guide the groups in bringing it together to solve equations involving exponentials and the change of base formula. Have groups post answers on the board.

$2^{-1} = .5$

$2^{0} = 1$

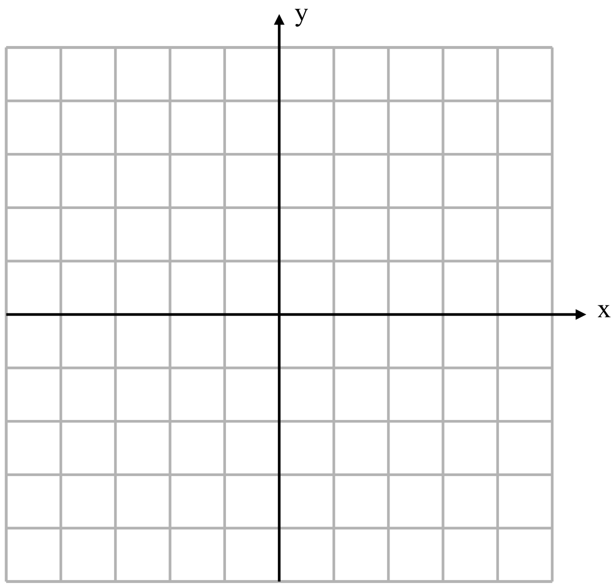
$2^{1} = 2$

$2^{3} = 8$

1. $2^{?} = 64 \quad ? = 6$

$2^{x} = 256 \quad x = 8$

Now graph $y = 2^{x}$



$3^{-1} = 1/3$

$3^{0} = 1$

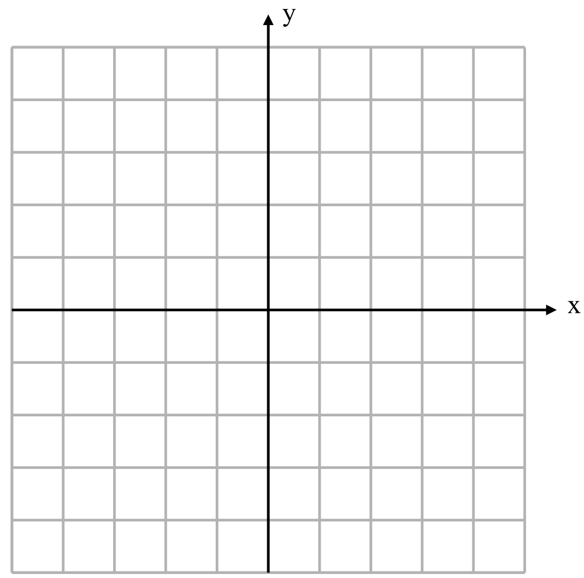
$3^{2} = 9$

$3^{4} = 81$

2. $3^{?} = 27 \quad ? = 3$

$3^{x} = 6561 \quad x = 8$

Now graph $y = 3^{x}$



$\left(\frac{1}{2}\right)^{-1} = 2$ $\left(\frac{1}{2}\right)^{0} = 1$

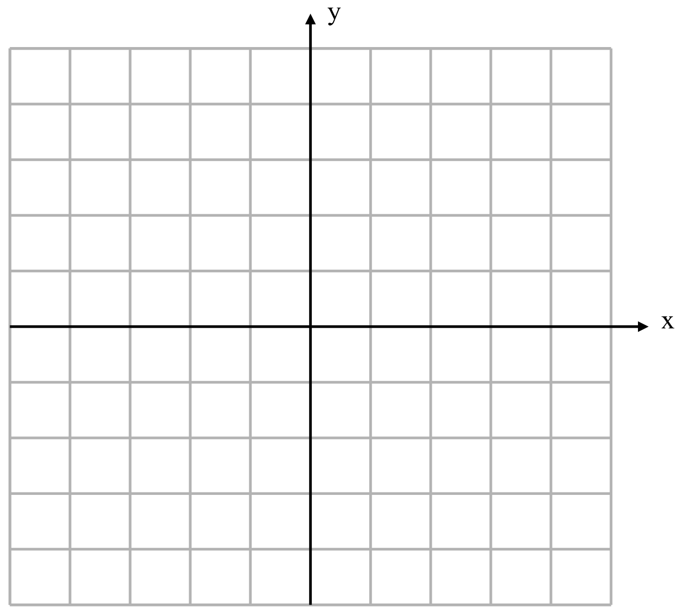
$\left(\frac{1}{2}\right)^{2} = 1/4$ $\left(\frac{1}{2}\right)^{4} = 1/16$

3.

$\left(\frac{1}{2}\right)^{?} = \frac{1}{64}$ $\left(\frac{1}{2}\right)^{x} = \frac{1}{512}$ $x = 9$

$? = 6$

Now graph $y = \left(\frac{1}{2}\right)^{x}$



Where do logs come from?

4. $\log_{10} 10 =$ 1

5. $\log_{10} 100 =$ 2

6. $\log_{10} 1000 =$ 3

Do you see the pattern for a quick way to evaluate each of the above logs?

Look for the observation that when the base and value inside log are related by a power the power is the result of the log

$\log_{10} 1000 = 3$ would be written as....

$10^{(\ 3\ )} = (\ 1000\ \ )$

and $\log_b y = x$ would be written as....

$b^{(x)} = y$

$10^x = y$ could be written $\log_{10}(\ y\ ) = (\ x\ )$

What about other bases rather than just base 10.

Use a calculator to find the following values: (**Round answers to thousandths**)

1. $\log_{10}(-10) =$ und
2. $\log_{10}(-5) =$ und
3. $\log_{10}(\frac{-1}{2}) =$ und
4. $\log_{10}(-.25) =$ und
5. $\log_{10} 0 =$ und
6. $\log_{10} 1 =$ 0
7. $\log_{10} 2 =$ .301
8. $\log_{10} 3 =$ .477
9. $\log_{10} 4 =$ .602
10. $\log_{10} 5 =$ .699
11. $\log_{10} 6 =$ .778
12. $\log_{10} 8 =$ .903
13. $\log_{10} 10 =$ 1
14. $\log_{10} 100 =$ 2
15. $\log_{10} 1000 =$ 3
16. $\log_{10} 10000 =$ 4
17. $\log_{10} \frac{1}{2} =$ -.301
18. $\log_{10} .1 =$ $\log_{10} \frac{1}{10} =$ -1
19. $\log_{10} .01 =$ $\log_{10} \frac{1}{100} =$ -2
20. $\log_{10} .001 =$ $\log_{10} \frac{1}{1000} =$ -3

$\log_e$ and $\ln$ are the same thing. The calculator has an ln button to use. Find each and round to nearest thousandth.

21. $\ln(-10) =$ und
22. $\ln(-2) =$ und
23. $\ln(-.25) =$ und
24. $\ln 0 =$ und
25. $\ln 1 =$ 0
26. $\ln(2.71828) =$ 1
27. $\ln e^1 =$ 1
28. $\ln e^5 =$ 5

Observations from the work above. Do you see any patterns?

A. Domain and Range of log and ln

B. log 1 and ln1 = 0

C. The sum of two logs = the log of the product

D. The difference of two logs = the log of the quotient

E. log ½ = -log2

F. e = 2.71828

G. Powers within log can come out in front of log

H. ln acts the same as log

Try writing some properties/rules for logs using what you observed. Use constants a and b rather than specific numbers.

A. log 1 and ln 1 both equal 0

B. log (ab) = log a + log b and same for ln

C. log (a/b) = log a – log b and same for ln

D. $\log_a a^b = b$

E. $\log a^b = b \log a$

Now try some examples with variables:

1. $\ln(xy) =$

ln x + ln y

2. $\log(x^3) =$

3log x

3. $\log(x^3y^2) =$

3log x + 2 log y

4. $\log(\frac{2x}{y}) =$

log2 + log x – log y

Welcome to 1614. That's when John Napier invented logarithms like we just did.

Some other problems (calculator needed)

1. $10^x = 12.75$

$\log_{10} 12.75 = x$ and solve

2. $e^x = 19.5$

$\ln 19.5 = x$ and solve

3. $3^x = 191.2$

$\log_3 191.2 = x$ Problem here

What problem occurs with #3 above? We need a way to change the base.

Take $3^x = 191.2$ and solve for x.

$$\log 3^x = \log 191.2$$
$$x \log 3 = \log 191.2$$
$$x = \frac{\log 191.2}{\log 3}$$

This is the change of base formula. Discuss it and how ln would work as well.

Student notes:

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1. Form groups.
2. Complete page 1, discuss as a class.
3. Complete page 2, discuss as a class.
4. Complete page 3, discuss as a class.
5. Complete page 4, discuss as a class.
6. Complete page 5, discuss as a class.

$2^{-1} =$

$2^{0} =$

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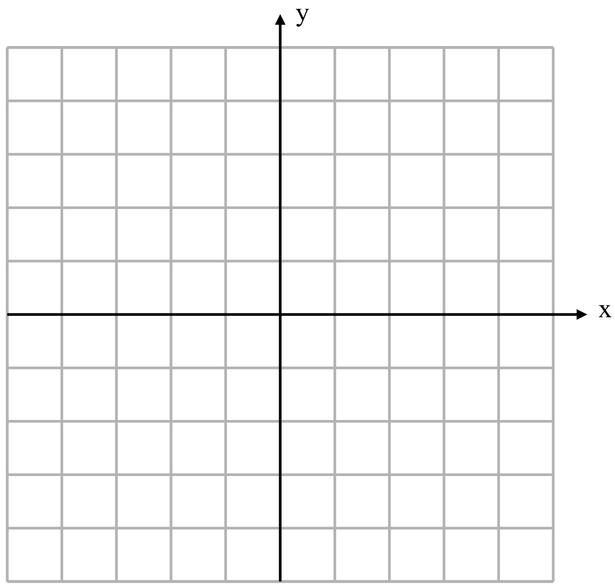
$2^{3} =$

1. $2^{?} = 64$

$2^{x} = 256 \quad x =$

Now graph $y = 2^{x}$



$3^{-1} =$

$3^{0} =$

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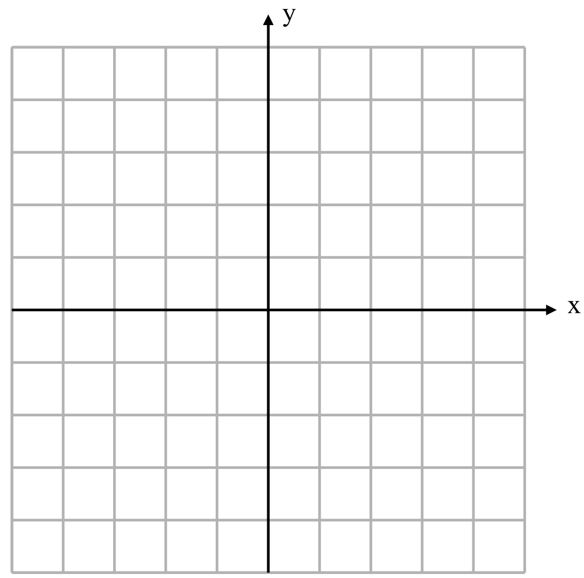

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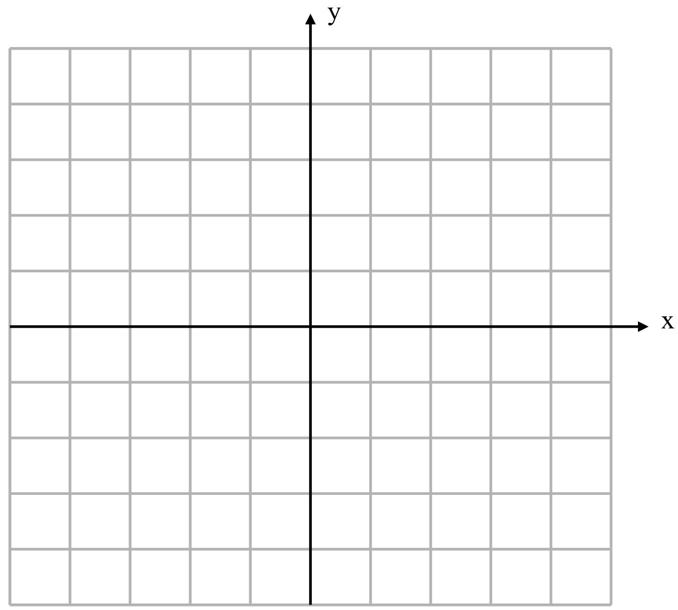

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5. $\log_{10} 0 =$ 6. $\log_{10} 1 =$ 7. $\log_{10} 2 =$ 8. $\log_{10} 3 =$

9. $\log_{10} 4 =$ 10. $\log_{10} 5 =$ 11. $\log_{10} 6 =$ 12. $\log_{10} 8 =$

13. $\log_{10} 10 =$ 14. $\log_{10} 100 =$ 15. $\log_{10} 1000 =$ 16. $\log_{10} 10000 =$

17. $\log_{10} \frac{1}{2} =$ 18. $\log_{10} .1 =$ $\log_{10} \frac{1}{10} =$ 19. $\log_{10} .01 =$ $\log_{10} \frac{1}{100} =$ 20. $\log_{10} .001 =$ $\log_{10} \frac{1}{1000} =$

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25. $\ln 1 =$ 26. $\ln(2.71828) =$ 27. $\ln e^1 =$ 28. $\ln e^5 =$

Observations from the work above. Do you see any patterns?

A.

B.

C.

D.

E.

F.

G.

H.

Try writing some properties/rules for logs using what you observed. Use constants a and b rather than specific numbers.

A.

B.

C.

D.

E.

Now try some examples with variables:

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2. $\log(x^3) =$

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