Lesson 5: Right Triangles in Construction

PREREQUISITE ASSUMPTIONS

Before beginning this lesson, students should

- Know that a right angle is a 90 degree angle
- Know that a right triangle has one 90 degree angle

MATERIALS NEEDED

- Sets of masonite strips cut to 1-2-3-4-5-6 units (one set per group)
- Framing Square
- Grid paper
- Rulers
- Scissors
- Colored pencils or pens

COMPETENCIES

From Unit 1-Use of Scientific Calculator

A. Perform basic mathematical operations

A.5. you perform exponentiation to powers of 2 and 3 applied to whole numbers

B. Apply basic mathematical operations in solving word problems

B.1. you translate a verbally stated problem into performing an equivalent computation

B.2. you interpret the computed answer to a word problem

B.3. you check the reasonableness of a computed answer to a word problem

Unit 5. Measurement

A. Perform calculations with quantities having units of measure: inches, inches squared, inches cubed, feet, feet squared and feet cubed only.

A.1. you perform addition, subtraction, multiplication, division, exponentiation to powers of 2 and 3, square root taking or combinations of these operations for quantities expressed as measurements

A.2. you expresses the answer with the appropriate units

A.3. you use a scientific calculator to compute the answer to problems involving measured quantities

B.3. you convert area measurements to different square and cubic units of length measure

Unit 6. Applied Plane Geometry

C. Perform calculations using the Pythagorean Theorem

C.1. you use the Pythagorean Theorem to solve for any missing side of a right triangle

C.3. you use a scientific calculator to perform calculations involving the Pythagorean Theorem

D. Calculate perimeters and areas of closed planar figures

D.7. you compute the area of a circle given its radius or diameter

Notes to Self

- One thing I want to do during this lesson ….
- One thing I want to pay attention in my students' thinking …
- One connection or idea I want to remember …

Suggested Timeline

Duration **115 minutes total**	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
15 minutes	Questions 1 - 3	Small groups
10 minutes	Discuss 2 and 3	Whole class
12 minutes	Lead up to Questions 4 – 6 and Question 4	Instructor led and small groups
12 minutes	Questions 5 - 6	Small groups
8 minutes	Discuss 4 - 6	Whole class
5 minutes	Question 7 - 8	Small groups
10 minutes	Discuss 7 - 8	Whole class
5 minutes	Question 9- 10	Small groups
3 minutes	Question 9 - 10	Whole class
12 minutes	Questions 11 - 13	Small groups
18 minutes	Discuss 11 and 13	Whole class
5 minutes	Making connections	Instructor led

[Student Day 1 Handout]

SPECIFIC OBJECTIVES

Use Pythagorean Theorem to determine the missing side of a right triangle and to create a 90 degree angle.

By the end of this lesson you will understand that...

- Pythagorean Theorem is a formula that allows you to find the missing side of a right triangle
- A right triangle has one 90 degree angle
- Right triangles are used in construction to create/confirm right angles
- A Pythagorean Triple is a set of whole numbers that make the Pythagorean Theorem true

By the end of this lesson you will be able to...

- Use square roots to solve the Pythagorean Theorem for a missing side of a triangle
- Use a Pythagorean Triple to confirm or create an angle that is exactly 90 degrees.

PROBLEM SITUATION #1 : Exploring Right Angles

Instructor reads the problem situation aloud to the students and then takes them out to the shop. The masonite strips should be pre-setup on the benches and each group gets its own bench.

Students are given 6 lengths of masonite strips with brads or nails to secure them together. The lengths are 1 unit, 2 units, 3 units, 4 units, 5 units and 6 units. Ideally, each group has a different 'unit' for their set of lumber. The lumber should have lines marking it to show the number of units on each piece.

In this problem situation, you will be going into the shop in your group and working together to figure out how to connect 3 of the pieces of masonite provided, to create a right (90°) triangle. You may use a framing square when you are done to verify your angle is 90 degrees.

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
15 minutes 10 minutes	Questions 1 – 3 Discuss 2 and 3	Small groups Whole class

Notes: The goal here is for every group to create a 3-4-5 right triangle. Since each group has different lengths for their base unit, lots of opportunities to talk about similar triangles and multiples later.

Possible areas for errors:

- Groups try to use more than 3 of the strips to make a triangle
- Group try to use the framing square to create the 90 degree angle first
- Groups don't use the framing square correctly and thus make a triangle that is not a right triangle

After each group has answered questions 2 and 3, Instructor leads a conversation to make sure all students recognize that their masonite strips have unit markings on them, just with different base units.

1. Ask students to identify the units for their triangles
2. Ask students how they indicated which angle in their drawing was the right angle (they should have put a box in that corner. If they didn't, have them do it during the discussion).
3. Make sure they labeled their sketches with units
4. Make sure everyone has '3-4-5' as units

Before returning to the classroom, tell students: ***"now I want you to measure your masonite strips (from brad to brad) and write those actual lengths on your sketches. Once you are done you can clean up your triangles and return to the classroom."***

1. Use 3 of the pieces to create a right angle. Sketch the shape you created and label it with as much information as you know.

Solution: Each group should sketch a 3-4-5 right triangle.

2. Once you see another group done with their work, check out their shape and compare it to yours.
    a. Describe all of the ways that the shapes are similar.

    b. Describe all of the ways that the shapes are different.

3. Repeat 2a and 2b with another group.
    a. Similarities

    b. Differences

Similarities: notice that everyone used the 3-4 and 5 unit pieces of lumber and created a right triangle Differences: the sizes of the triangles are different.

Now, let your instructor know you are done and wait for your instructor to lead a whole class conversation…

PROBLEM SITUATION #2 : Pythagoras

In this part of the lesson, you will need to wait until your instructor gives you guidelines on next steps. Take notes on what is shared in the space below:

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
12 minutes	Lead up to Questions 4 – 6 Question 4	Instructor led Small groups
Notes: detailed notes for the instructor led activity are provided below. Question 4 instructions are embedded in the activity.		

Instructor draws a right triangle on the board and labels it… providing definitions if necessary (like perpendicular). Have students go back to #1 and make sure students also have a right triangle on their paper. They can add the vocab/labels to their drawing.

Instructor writes the Pythagorean Theorem on the board:
$a^2 + b^2 = c^2$
Ask students: "***raise your hand if you recognize this formula.***"
Ask students: ***"keep their hand up if they know what it's called"***
Ask students: ***"keep their hand up if they know what it's for"***

Call on one of the students that still has their hand up to name it and explain it
Looking for: It's the Pythagorean Theorem. If the three sides of a triangle make the equation true, we know it is a right triangle.

Now, tell students to think about when they first constructed the triangles they made and ask ***"what units did everyone originally have for their triangles (before measuring actual lengths)?"***

Label the triangle on the board with the dimensions up on the board. Good opportunity to ask which dimension is the hypotenuse…

Demonstrate how we really know it's a right triangle… Put a, b and c on the triangle above the 3, 4 and 5. Then sub into the formula… because 3^2 + 4^2 = 5^2 … those 3 numbers make the equation true. Any 3 numbers that make the equation true will form a right triangle.
But WHY does the formula work?

Announce: ***"You are going to do a bit of geometry to see how it connects to the Pythagorean Theorem…"***

SET UP FOR THE PAPER CUTTING ACTIVITY:
Have one student from each group come up to the front and get grid paper and scissors for their group (enough so each student gets their own)
Demonstrate constructing the squares (tell students to follow along) by taking a piece of graph paper

Commented [BB1]: At some point in this lesson we need to teach them how to use the square and quare root functions of their calculators.

on the Doc Cam and:

1. Write the 3 dimensions (3-4-5) at the top of the paper and then state that you are going to draw a line of length '3 units.' Remind students that the 'unit' can be different, as long as it holds to their triangle sides. So, use 1" = 1 unit (4 squares on the grid paper). Encourage students to pick their own 'unit' for their drawing.
2. Tell students ***"write down the units you are using for your squares in question 4."***
3. Draw a line of length a and then extend the shape into a square and ask the students what the area of the square is… write a^2 underneath the square.
    a. (optional) As a refresher from previous math classes, when you draw a square on the board showing right angle marks and all sides lengths a units ask: What's the name of this shape? What makes it a square? Is it JUST that all sides are equal length (sketch a rhombus). What is its size--how big is it--what is its measure? This will remind students that a^2 represents the area of a square that has a side length of a.
4. Repeat with b and c next to a, so that the squares line up in a row and you can rewrite the Pythagorean Theorem under each square.
5. NOTE: do NOT draw the right triangle and extend the squares off the triangle during this demonstration, it will cause confusion for how to cut the squares out on the graph paper and will likely double the time it takes to do this activity!

After demonstrating the creation of the 3 squares, tell students ***"now move on to #5. Be sure to talk and come up with at least one strategy. Folks in the same group can use different strategies if you want. Try to come up with as many different strategies as you can think of before you move on to question 6."***

4. What units did you decide to use to construct your squares?

Some possible answers: 1" = 1 unit. The length of 2 grid squares = 1 unit. 2" = 1 unit

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
12 minutes 8 minutes	Questions 5 - 6 Discuss 5 - 6	Small groups Whole class

Notes: Makes sure students stay on task. Make sure students are actively engaged in conversation for #5. If a group isn't talking, go up and ask how it is going. If they say they don't know where to start, ask them to look at the 3 squares and say ***"is there any way you can make the two smaller squares fit into the larger square somehow…"***

For #6, Instructor should cruise the room while the students work and seek out different strategies for

showing that the two smaller squares = the larger square.
Redirect students that just do the calculations with numbers that we want them to also show it with geometry.
Pick out at least two different methods and ask the students if they will explain to the class what they did. If they say okay, then when it comes time to discuss, ask the pre-selected students to go up to the doc cam to explain what they did

Be sure, in the discussion, that it is clearly written, shown that:
3^2 + 4^2 = 5^2 because…. 9 + 16 = 25 works. It makes the equation true.

(optional) After completing the conversation, show the Project Mathematics Video from 12:25 - 15:00

5. Discuss in your group how you can use your 3 squares to prove that the areas of the two smaller squares = the area of the larger square ($a^2 + b^2 = c^2$). Write down your selected strategy below.

Possible solutions:
Students could cut the two smaller squares out of the paper and cut them into pieces so they fill the grid in the largest square.
Students could use colored pencils or markers to show that 9 squares from a and 16 squares from b fill up the 25 squares in c
Students could cut up the larger square and show that the pieces it is cut into exactly fit into the two smaller squares

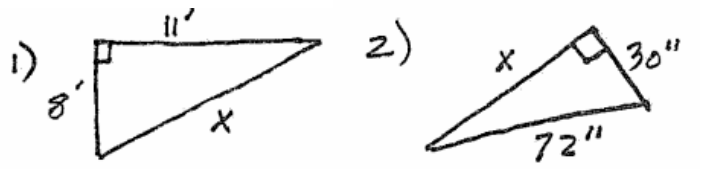
6. Now, use your strategy you described in #5. Did it work? If not, why not?

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
5 minutes 10 minutes	Question 7 - 8 Discuss 7 - 8	Small groups Whole class
Notes: The goal here is get students to understand that ANY set of units that are 3-4-5 multiples works to create a right triangle. The goal of #8 is to gauge the class's experience with the Pythagorean		

Theorem in general.

Have one person from each group put the calculations on a white board.
Ask… ***"why does it work for ALL of these different measurements?"***
Looking for a student to point out that all of them are 'multiples of 3-4-5'
Demonstrate… 18" is 3 x 6", 24" is 4 x 6", 30" is 5 x 6"… do that with a couple. Ask ***"could have used 6' instead of 6"?"***

After showing the Take-Aways from #7, have students share their answers to #8 and put up the following two drawings on the white board. Tell students ***"copy these drawings into your lesson and work with your group to try and find the missing side length."***



Cruise the room while they are working. If a lot (half or more) of the class seems stumped, STOP the work and turn these two problems into a demonstration.

Even if groups seem like they are working okay, still fully demonstrate solving these two problems using the square root button on the calculator and clearly write the steps on the board. Make sure students write them down.
Also ensure that students know the vocabulary of 'perpendicular' and making sure things are 'square.'

7. Now take the measurements you wrote down for the triangle you created in problem situation #1 and substitute those 3 numbers into the Pythagorean Theorem. Calculate and determine if they make the equation true. Show your work below.

Answers:
Each group will have a different equation depending on their base unit.
6" base unit: (18")^2 + (24")^2 = (30")^2 so 324 + 576 = 900 ? yes !
2" base unit: (6")^2 + (8")^2 = (10")^2 so 36 + 64 = 100 ? yes!
Etc…

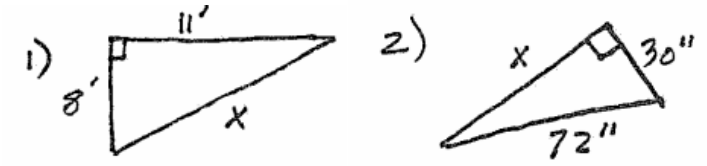
Take Away :
Similar triangles are triangles that have sides that are multiples of each other (same shape, different

size). We can use this fact to use the 3-4-5 triangle so that it works for our application... whether it is 3" – 4" and 5" or 6' – 8' and 10'.
By the way, the 3-4-5 set of numbers has a special name, it is a 'Pythagorean Triple.' That means, it is a set of whole numbers that makes the Pythagorean Theorem true. There are other Pythagorean Triples (like 5-12-13) that also exist!

8. What if you want to make a right angle, how can you use the Pythagorean Theorem to help you? Discuss in your group and write down your explanation below.

Answer: If you know two of the sides, you can use Pythagorean Theorem to find the 3rd side, then if you connect those 3 lengths, you know for sure the angle is a right angle.

Leave lots of space for this problem so there is room for the students to copy down the two drawings



Solutions:
1) 8^2 + 11^2 = x^2

PROBLEM SITUATION #3: Uses for the Pythagorean Theorem in construction

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)

5 minutes 3 minutes	Work Question 9 - 10 Discuss Question 9 - 10	Small groups Whole class
Notes: This should just take a few minutes. Do not let the discussion drag on. The experienced students will know lots of different ways. During the share-out, have each group provide one answer and then let students add on to the list. Pre-watch video of Allie Berenyi squaring a wall: https://youtu.be/VqAHXYeNeSM		

9. With your group, brainstorm as many ways a possible that you think you could use a 3-4-5 right triangle to help you on a construction site. Once all groups have shared their ideas, we will select one of the applications you came up for #10 below.

Possible answers: Squaring a wall or floor framing, ensuring the tiles laid for a flooring job are perpendicular, ensuring two walls are perpendicular to each other when framing a house, laying out ceiling grid, squaring up the concrete foundation wall to layout for the mudsill, squaring up footing forms, laying out for a building before excavation, lay out a rafter, calculate stringer lengths.

10. State the *best* way you think you could use a 3-4-5 right triangle for the task.

Answers: Likely that experienced students will say how to complete the task…
'draw an arc' is part of the work that should be explained.
Don't let the conversation get bogged down, since 11 – 13 has them go DO it.

Duration	**Activity** (Indicate question number)	**Suggested Structure** (Indicate group, whole class or individual work)
12 minutes 18 minutes	Work Questions 11 - 13 Discuss Questions 11 - 13	Small groups Whole class
Notes: Students now have to apply the idea of using the Pythagorean Theorem in a real setting. They will walk out to the shop in their groups. Each group gets a different line of length x. Instructions: **"You are going to snap a chalk line for a wall that is square to one that has already been snapped on the floor for you. Your wall will start at one end of the line on the floor. You may choose the length of that wall, we'll call it length b. *Write your a and b values in #11. Then move on and do #12 and #13. Be sure to discuss your strategy for #13 with your group before you start it!*"** A line of a length is marked onto the floor (one for each group). - make a multiples of 3 or 4		

The group selects their own length for the other leg (b) - see if any groups figure out they can make a 3-4-5 right triangle
They then use Pythagorean theorem to calculate the diagonal (c)

For #13 - A person with two tape measures stands at the b-c vertex. One person takes one tape to the a-b vertex and another person takes the other tape to the a-c vertex to see if they are correct.

If the students are struggling, provide a tip… one person can hold two tape-measures and there are 3 of you, so one at each corner…

Each group must demonstrate #13 to another group or the Instructor. After they do, let students return to the classroom to start the homework. Once everyone is done, point out interesting techniques people used to create their squared walls.

End by showing the video of Allie Berenyi squaring a wall: https://youtu.be/VqAHXYeNeSM

11. Let's go out to the shop and practice squaring a wall using 2 tape measures. Once you are in the shop, listen to the instructions from your instructor and then write down your values of a and b below:

    a= _____b = _____

12. Calculate your hypotenuse (diagonal) using the Pythagorean Theorem ($a^2 + b^2 = c^2$). Show your work below.

13. Check your work by measuring out your triangle. Did it work?

Take-aways:

- Make the two 'legs' as long as possible (maybe even the whole wall) to improve accuracy of the right angle
- Can use 3-4-5 multiples OR full length. Your choice

Commented [BB2]: Re: question 9

"Why would I want to make a right triangle?", asks the student. Rather than thinking about the right traingle in a vacuum, I think skip this question and jump right down to #9, the context for making right triangles and thus, using the pythag theorem.

Commented [SP3R2]: she means #8

Commented [SA4R2]: are you sure?

Commented [BB5]: We could have each group check another group's work. This might take more time than having the instructor checking them.

Commented [SP6R5]: good idea

MAKING CONNECTIONS

Record the important mathematical ideas from the discussion.

- A 3-4-5 triangle always makes a right triangle because it makes the Pythagorean theorem true
- Any triangle that makes the Pythagorean theorem true is a right triangle
- The Pythagorean theorem is $a^2 + b^2 = c^2$
- Using right triangles in construction allows the builder to ensure that things they are building are square, containing a right angle and a pair of perpendicular lines.

Commented [CP7]: Great video! YouTube STAR!

PRACTICE

Pg 121 2, 4
Pg 122 5
Pg 128 2, 5
Pg 129 7, 8, 10
Pg 130 11, 12
Pg 131 19